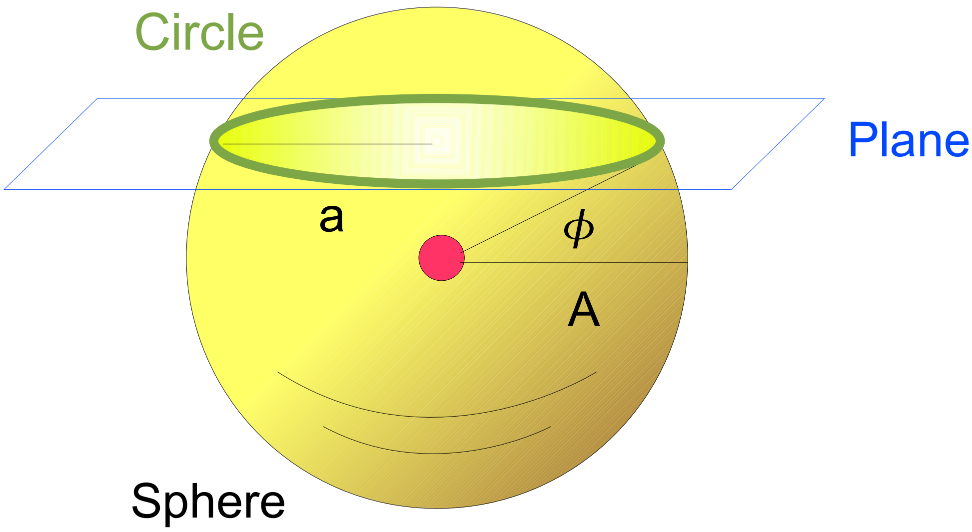
Dieter Egger

About a Special Metric



Dark Matter and Dark Energy transmogrified

Preface

Assuming an expanding and then contracting universe described by a suitable dynamic metric tensor we shall calculate the corresponding energy-impulse tensor out of Einstein's field equations.

For the sake of clarity we first treat a 2-dim space-time. In this simple case you may imagine the chosen metric as resulting from the intersection of the 2-dim surface of a 3-dim sphere with an evenly moving 2-dim plane. This 1-dim living space changes its size due to the movement of the plane through the sphere and is first expanding and then contracting.

To come closer to our „reality" the number of space-like dimensions will be increased to three. So we shall have the situation where the 4-dim hypersurface of a 5-dim hypersphere gets intersected with an evenly moving 4-dim hyperplane to provide a 3-dim living room which is expanding first and then contracting.

Some results gained by numerical integration reveal the interesting behaviour of light when being transmitted in the far past and reaching us today. It's quite astonishing that a closed universe may be faking an accelerated expanding one. And it's an astounding fact too that it may also „hide" tremendous amounts of gravitationally acting masses which may never get seen.

Please send questions and comments to „dr.egger at alice.de"

Dieter Egger, Munich, 2013-06-10

About the Author

He was responsible for teaching object-oriented programming to master students of „geodesy and geoinformation" at the Technische Universität München for about 18 years. And he was also teaching some fundamentals of astronomy and cosmology. And always had in mind that our universe is expanding at a decreasing rate, comes to a standstill one day and then starts contracting faster and faster until the „big crunch" will finish the current era and create a new one via another „big bang" and so on ...

Using computers and software for numerical integration and symbolic formula manipulation allowed to do all the extensive calculations arising from the intention to show another way than „dark energy" and „dark matter" to explain the framework of our universe.

Contents

Fundamentals

Idea

A plane „glides“ evenly through a sphere and cuts out a perimeter of the surface of the sphere which serves as a 1-dim living room for 1-dim beings. Its evolution is given and even forced by the continually (or discretely) moving plane.

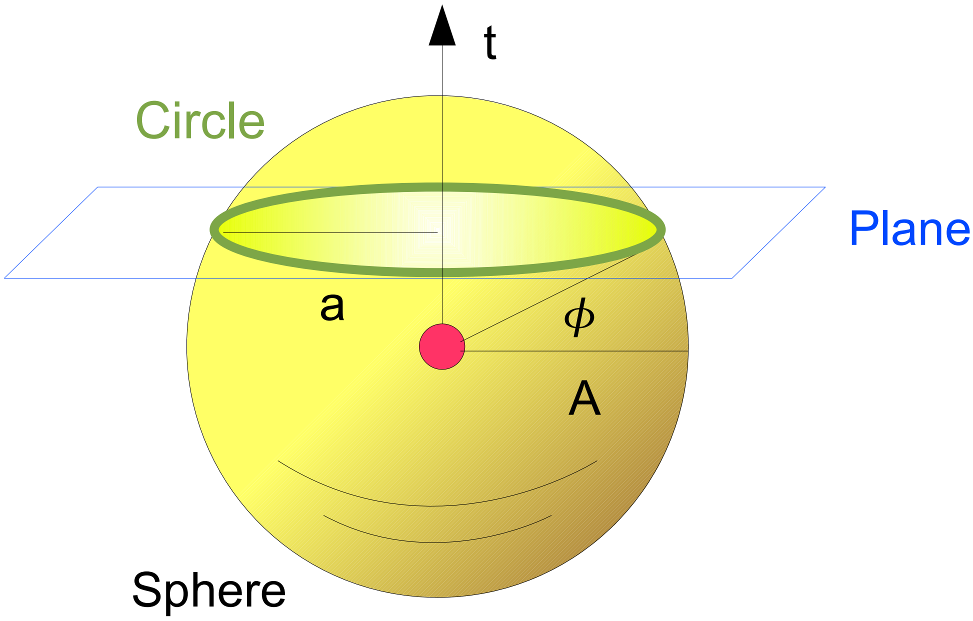


Figure 1: Plane gliding evenly through a sphere and thus providing a 1-dim perimeter of the circle as a dynamic1-dim living room.

Due to the evenly moving plane the angle ϕ is changing like

$\phi = \arcsin(t)$ resp. $\sin\phi = t$, with t linearly increasing from -1 to $+1$.

t is a free „cosmic“ parameter responsible for the changeability within the 1-dim world.

The extension to more spatial dimensions is quite easy:

A (n-1)-dim hyperplane „glides“ evenly through the (n-1)-dim hypersurface of a n-dim hypersphere (radius A) and provides as intersections a series of (n-2)-dim hypersurfaces (with actual radii a) serving as a dynamic „living room“ for (n-2)-dim „beings“.

Characteristic Parameters

Based on t we calculate some characteristic data. They do not yet have units.

parameter	as function of t	using a(t)
size (radius)	$a(t)=A\sqrt{1-t^2}$	$a(t)$
velocity	$\frac{da}{dt}=\frac{-At}{\sqrt{1-t^2}}=\frac{-A\sqrt{1-t^2}t}{1-t^2}$	$\frac{da}{dt}=-a(t)t\frac{1}{(1-t^2)}$
acceleration	$\frac{d^2a}{dt^2}=-\frac{A}{\sqrt{1-t^2}(1-t^2)}=-\frac{A^2}{A\sqrt{1-t^2}(1-t^2)}$	$\frac{d^2a}{dt^2}=-\frac{A^2}{a(t)}\frac{1}{(1-t^2)}$

Table 1: Characteristic parameters of the 1-dimensional universe. „A“ represents the maximum size and is given by the radius of the 3-dimensional sphere.

Please note that the acceleration is never vanishing and always negative. So two observers get the impression that there is always an attractive force acting upon them.

Let's try to obtain some numerical values for $t=0$ when expansion has reached its maximum. To achieve this we first have to assign common units to the parameters (preferably SI-units):

$a(t[s])[m]=A[m]\sqrt{1-\frac{(t[s])^2}{(t_u[s])^2}}$, with t linearly increasing from $-t_u$ to $+t_u$ and t_u representing the time to reach maximum expansion gives the actual radius,

$\frac{da}{dt}\left[\frac{m}{s}\right]=-a(t)[m]t[s]\frac{1}{(t_u^2-t^2)[s^2]}$ is the expansion rate

and finally

$\frac{d^2a}{dt^2}\left[\frac{m}{s^2}\right]=-\frac{(A[m])^2}{a(t)[m]}\frac{1}{(t_u^2-t^2)[s^2]}$ the appropriate acceleration.

Assuming that we're about $t=0$ and thus about the maximum expansion size we may introduce todays' radius of the universe

$$A \approx 13{,}7\cdot 10^9\, ly \approx 1{,}296\cdot 10^{26}\, m$$

and todays' age of the universe

$t_u \approx 13{,}7\cdot 10^9\, a \approx 4{,}323\cdot 10^{17}\, s$ and thus $(t_u^2-t^2)\approx 1{,}869\cdot 10^{35}[s^2]$

to calculate nowadays size, expansion speed and expansion acceleration

$a(t=0)=A[m]$, $\frac{da}{dt}=0\left[\frac{m}{s}\right]$ and $\frac{d^2a}{dt^2}=-6.935\cdot 10^{-10}\left[\frac{m}{s^2}\right]$

To explain the acceleration in a flat space-time one had to introduce a mass responsible for the attraction.

In a distance of $a(t=0)=A[m]$ the mass had to amount to $m\approx 1{,}745\cdot 10^{53}[kg]$ correspon-ding to about $8{,}725\cdot 10^{22}$ suns or $8{,}725\cdot 10^{11}$ galaxies. But only a small part of about $1\cdot 10^{11}$ galaxies containing about $1\cdot 10^{11}$ sun masses each has been detected so far. The rest goes into the realm of „dark things“, here into „dark matter“.

Metric

Path length in general

The infinitesimal element of the path resp. the square of it is

$$(ds)^2=g_{\mu\nu}dx^{\mu}dx^{\nu}$$

giving the complete path length

$$s=\int_{t_A}^{t_B}\sqrt{g_{\mu\nu}\frac{dx^{\mu}}{dt}\frac{dx^{\nu}}{dt}}\;dt$$

with $g_{\mu\nu}$ metric or pseudo metric.

Metric of the surface of a sphere

We describe the surface of the 3-dim sphere with radius *A* as a function in 3-dim space (longitude λ , latitude ϕ) and take into account the dynamic $\sin\phi=t$:

$$f:t,\lambda\rightarrow A\begin{pmatrix}\sqrt{1-t^2}\cos\lambda\\ \sqrt{1-t^2}\sin\lambda\\ t\end{pmatrix}$$

for calculating the components of the metric tensor

$$g_{00}(t,\lambda)=\frac{\partial f}{\partial t}\frac{\partial f}{\partial t}=\frac{A^2}{1-t^2}$$

$$g_{01}=g_{10}=\frac{\partial f}{\partial t}\frac{\partial f}{\partial\lambda}=\frac{\partial f}{\partial\lambda}\frac{\partial f}{\partial t}=0$$

and

$$g_{11}(t,\lambda)=\frac{\partial f(t,\lambda)}{\partial\lambda}\frac{\partial f(t,\lambda)}{\partial\lambda}=A^2(1-t^2)$$

Metric tensor

Now having the (covariant) metric tensor (cellar indices)

$$g_{\mu\nu}(t,\lambda)=\begin{pmatrix} \frac{A^2}{1-t^2} & 0 \\ 0 & A^2(1-t^2) \end{pmatrix}$$

with determinant $det(g_{\mu\nu})=A^4$ it's easy to get the contravariant metric tensor (roof indices)

$$g^{\mu\nu}(t,\lambda)=\begin{pmatrix} \frac{1-t^2}{A^2} & 0 \\ 0 & \frac{1}{A^2(1-t^2)} \end{pmatrix} .$$

Path length in particular

The infinitesimal element of the path resp. the square of it is

$$(ds)^2=g_{\mu\nu}dx^\mu dx^\nu=g_{00}\,dt\,dt+g_{11}\,d\lambda\,d\lambda=\frac{A^2}{1-t^2}dt^2+A^2(1-t^2)d\lambda^2$$

giving the complete path length

$$s=\int_{t_A}^{t_B}\sqrt{g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}\,dt=\int_{t_A}^{t_B}\sqrt{\frac{A^2}{1-t^2}+A^2(1-t^2)\left(\frac{d\lambda}{dt}\right)^2}\,dt$$

with $g_{\mu\nu}(t,\lambda)$ inserted. Although having no movement in λ -direction, this gives $s=A\arcsin t=A\phi$ due to the unavoidable movement in t -direction.

Christoffel symbols

With the partial derivatives of the metric coefficients

$$g_{00,0}=g_{tt,t}=A^2\frac{\partial}{\partial t}(1-t^2)^{-1}=-A^2(1-t^2)^{-2}(-2t)=2A^2\frac{t}{(1-t^2)^2}$$

$$g_{00,1}=g_{tt,\lambda}=0\ ,\quad g_{01,0}=g_{t\lambda,t}=0\ ,\quad g_{01,1}=g_{t\lambda,\lambda}=0\ ,\quad g_{10,0}=g_{\lambda t,t}=0\ ,\quad g_{10,1}=g_{\lambda t,\lambda}=0$$

$$g_{11,0}=g_{\lambda\lambda,t}=-2A^2t$$

$$g_{11,1}=g_{\lambda\lambda,\lambda}=0$$

and the rule $\Gamma^{\sigma}_{\mu\nu}=\frac{1}{2}g^{\sigma\rho}\left(g_{\rho\mu,\nu}+g_{\rho\nu,\mu}-g_{\mu\nu,\rho}\right)$ we may calculate the Christoffel symbols straightforwardly

$$\Gamma^0_{00}=\frac{1}{2}g^{0\rho}\left(g_{\rho0,0}+g_{\rho0,0}-g_{00,\rho}\right)=\frac{1}{2}[g^{00}\left(g_{00,0}+g_{00,0}-g_{00,0}\right)+g^{01}\left(g_{10,0}+g_{10,0}-g_{00,1}\right)]$$

$$\Gamma^0_{00}=\frac{1}{2}g^{00}g_{00,0}=\frac{1}{2}\ \frac{1-t^2}{A^2}\ 2A^2\frac{t}{(1-t^2)^2}=\ \frac{t}{1-t^2}$$

$$\Gamma^1_{00}=\frac{1}{2}g^{1\rho}\left(g_{\rho0,0}+g_{\rho0,0}-g_{00,\rho}\right)=\frac{1}{2}[g^{10}\left(g_{00,0}+g_{00,0}-g_{00,0}\right)+g^{11}\left(g_{10,0}+g_{10,0}-g_{00,1}\right)]=\ 0$$

$$\Gamma^0_{01}=\frac{1}{2}g^{0\rho}\left(g_{\rho0,1}+g_{\rho1,0}-g_{01,\rho}\right)=\frac{1}{2}[g^{00}\left(g_{00,1}+g_{01,0}-g_{01,0}\right)+g^{01}\left(g_{10,1}+g_{11,0}-g_{01,1}\right)]=\ 0$$

$$\Gamma^1_{01}=\frac{1}{2}g^{1\rho}\left(g_{\rho0,1}+g_{\rho1,0}-g_{01,\rho}\right)=\frac{1}{2}[g^{10}\left(g_{00,1}+g_{01,0}-g_{01,0}\right)+g^{11}\left(g_{10,1}+g_{11,0}-g_{01,1}\right)]$$

$$\Gamma^1_{01}=\frac{1}{2}g^{11}g_{11,0}=\frac{1}{2}\frac{1}{A^2(1-t^2)}\ (-2A^2t)=\ -\frac{t}{1-t^2}$$

$$\Gamma^0_{10}=\ 0$$

$$\Gamma^1_{10}=\Gamma^1_{01}=\ -\frac{t}{1-t^2}$$

$$\Gamma^0_{11}=\frac{1}{2}g^{0\rho}\left(g_{\rho1,1}+g_{\rho1,1}-g_{11,\rho}\right)=\frac{1}{2}[g^{00}\left(g_{01,1}+g_{01,1}-g_{11,0}\right)+g^{01}\left(g_{11,1}+g_{11,1}-g_{11,1}\right)]$$

$$\Gamma^0_{11}=\frac{1}{2}g^{00}(-g_{11,0})=\frac{1}{2}\ \frac{1-t^2}{A^2}\ (2A^2t)=\ (1-t^2)t$$

$$\Gamma^1_{11}=\frac{1}{2}g^{1\rho}\left(g_{\rho1,1}+g_{\rho1,1}-g_{11,\rho}\right)=\frac{1}{2}[g^{10}\left(g_{01,1}+g_{01,1}-g_{11,0}\right)+g^{11}\left(g_{11,1}+g_{11,1}-g_{11,1}\right)]=\ 0$$

Covariant derivative

Having a vector $B^{\mu}=\begin{pmatrix}B^0\\B^1\end{pmatrix}$ and the rule for covariant derivation $B^{\mu}{}_{;\nu}=B^{\mu}{}_{,\nu}+\Gamma^{\mu}_{\nu\sigma}B^{\sigma}$ (please note the semicolon), with $B^{\mu}{}_{,\nu}=\frac{\partial B^{\mu}}{\partial x^{\nu}}$ depicting the usual partial derivative (please note the comma) , we get

$$B^0{}_{;0}=B^0{}_{,0}+\Gamma^0_{0\sigma}B^{\sigma}=B^0{}_{,0}+\Gamma^0_{00}B^0+\Gamma^0_{01}B^1=\ \frac{\partial B^0}{\partial t}+\frac{t}{1-t^2}\ B^0$$

$$B^0{}_{;1} = B^0{}_{,1} + \Gamma^0_{1\sigma} B^\sigma = B^0{}_{,1} + \Gamma^0_{10} B^0 + \Gamma^0_{11} B^1 = \frac{\partial B^0}{\partial \lambda} + (1-t^2) t \; B^1$$

$$B^1{}_{;0} = B^1{}_{,0} + \Gamma^1_{0\sigma} B^\sigma = B^1{}_{,0} + \Gamma^1_{00} B^0 + \Gamma^1_{01} B^1 = \frac{\partial B^1}{\partial t} - \frac{t}{1-t^2} \; B^1$$

$$B^1{}_{;1} = B^1{}_{,1} + \Gamma^1_{1\sigma} B^\sigma = B^1{}_{,1} + \Gamma^1_{10} B^0 + \Gamma^1_{11} B^1 = \frac{\partial B^1}{\partial \lambda} - \frac{t}{1-t^2} \; B^0$$

and for $B^\mu = \begin{pmatrix} B^0 \\ B^1 \end{pmatrix} = A \begin{pmatrix} \arcsin t \\ \sqrt{1-t^2} \;\; \lambda(t) \end{pmatrix}$ the result is

$$B^0{}_{;0} = \frac{A}{\sqrt{1-t^2}} + \frac{t}{1-t^2} \; B^0$$

$$B^0{}_{;1} = (1-t^2) t \; B^1$$

$$B^1{}_{;0} = -A \lambda \frac{t}{\sqrt{1-t^2}} + A \sqrt{1-t^2} \frac{d\lambda}{dt} - \frac{t}{1-t^2} \; B^1$$

$$B^1{}_{;1} = A \sqrt{1-t^2} - \frac{t}{1-t^2} \; B^0$$

Curvature tensor (Riemann tensor)

$R^\alpha_{\beta\mu\nu} = \partial_\mu \Gamma^\alpha_{\beta\nu} - \partial_\nu \Gamma^\alpha_{\beta\mu} + \Gamma^\alpha_{\sigma\mu} \Gamma^\sigma_{\beta\nu} - \Gamma^\alpha_{\sigma\nu} \Gamma^\sigma_{\beta\mu}$ according to [Schröder]

$R^m_{ikp} = \partial_p \Gamma^m_{ik} - \partial_k \Gamma^m_{ip} + \Gamma^r_{ik} \Gamma^m_{rp} - \Gamma^r_{ip} \Gamma^m_{rk}$ according to [Fließbach]

Reduce, an open software for symbolic formula manipulation (cf. appendix) gives according to [Fließbach] the only non-zero components

$$R^0_{101} = -(1-t^2) \;\; , \;\; R^0_{110} = +(1-t^2) \;\; , \;\; R^1_{001} = +\frac{1}{1-t^2} \;\; , \;\; R^1_{010} = -\frac{1}{1-t^2}$$

[Schröder] und [Fließbach] only differ in the sign of the curvature tensor. But that's only a matter of convention. We shall follow the opinion that a positive curvature should give a positive curvature scalar

Ricci tensor

$R_{\mu\nu} = R^\rho_{\mu\nu\rho} = -R^\rho_{\mu\rho\nu}$ according to [Schröder]

$R_{ij} = R^m_{imj}$ according to [Fließbach]

Reduce comes up with

$$R_{ij} = \begin{pmatrix} \frac{1}{1-t^2} & 0 \\ 0 & 1-t^2 \end{pmatrix}$$

and the

Curvature scalar

$$R = g^{\mu\nu} R_{\mu\nu}$$

resulting in

$$R = \frac{2}{A^2}$$

Einstein tensor

The Einstein tensor abbreviates the left hand side of the field equations:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{R}{2} g_{\mu\nu}$$ [Fließbach], [Schröder] in harmony

and gives for our special metric in the 2-dim space-time

$$G_{\mu\nu} = \begin{pmatrix} \frac{1}{1-t^2} & 0 \\ 0 & 1-t^2 \end{pmatrix} - \frac{1}{A^2} \begin{pmatrix} \frac{A^2}{1-t^2} & 0 \\ 0 & A^2(1-t^2) \end{pmatrix} = 0$$

a vanishing tensor.

Einstein's field equations

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

If the left hand side tensor vanishes in our 2-dim space-time then the right hand side tensor, the energy-impulse tensor vanishes too, meaning that our proposed metric does not need extra matter or energy or they just cancel out each other.

Equation of motion

$$\frac{d^2 x^\kappa}{d\tau^2} = -\Gamma^\kappa_{\mu\nu} \frac{d x^\mu}{d\tau} \frac{d x^\nu}{d\tau}$$

with τ proper time [Fließbach, Schröder] is valid for particles.

Photons do not have a proper time, so the equation of motion is written as

$\frac{d^2 x^\kappa}{dt^2} = -\Gamma^\kappa_{\mu\nu} \frac{d x^\mu}{dt} \frac{d x^\nu}{dt}$ with t now representing a parameter describing the path.

If we assume that per step along the direction of time (cf. Figure 1)

$$dx^0 = A\, d\phi = \frac{A}{\sqrt{1-t^2}}\, dt$$

(path along the meridian when the plane moves a distance dt through the sphere)

not more than the same amount of movement is allowed along the direction of space

$$dx^1 = d(a\lambda)$$

(path along the parallel of latitude)

we get for the maximum velocity (e.g. the velocity of light)

$$\frac{dx^1}{dt} = \pm \frac{dx^0}{dt}$$

with x^0 along increasing ϕ and x^1 along $\pm\lambda$.

It's a constant maximum velocity within the „living room“ perimeter, but a variable maximum velocity with respect to the surface of the sphere (t, λ) .

More dimensions

In this section we mainly follow the version of [Fließbach] but choose the signs according to a positive curvature scalar when the curvature is positive.

Curvature scalar

Dimension of space-time	Coordinates	Curvature scalar	Einstein tensor
2	t, λ_0	$\frac{2}{A^2}$	$G_{\mu\nu}=0$
3	$t, \lambda_{0,} \lambda_1$	$\frac{6}{A^2}$	$G_{\mu\nu}\neq 0$
4	$t, \lambda_{0,} \lambda_{1,} \lambda_2$	$\frac{12}{A^2}$	$G_{\mu\nu}\neq 0$

Table 2: Considering more than two dimensions for the space-time gives increasing values for the curvature scalar and non-vanishing Einstein tensors.

Einstein tensor

In 3- and 4-dimensional space-time the Einstein tensor does no longer vanish.

3-dim space-time

$$G_{\mu\nu} = \begin{pmatrix} \frac{2}{1-t^2} & 0 & 0 \\ 0 & 2(1-t^2) & 0 \\ 0 & 0 & 2(1-t^2)\cos^2\lambda_0 \end{pmatrix} - \frac{3}{A^2} \begin{pmatrix} \frac{A^2}{1-t^2} & 0 & 0 \\ 0 & A^2(1-t^2) & 0 \\ 0 & 0 & A^2(1-t^2)\cos^2\lambda_0 \end{pmatrix} =$$

combines to

$$G_{\mu\nu} = \begin{pmatrix} -\frac{1}{1-t^2} & 0 & 0 \\ 0 & -(1-t^2) & 0 \\ 0 & 0 & -(1-t^2)\cos^2\lambda_0 \end{pmatrix} \text{ in the } (t, \lambda_{0,} \lambda_1)-world$$

4-dim space-time

$$G_{\mu\nu} = \begin{pmatrix} -\frac{3}{1-t^2} & 0 & 0 & 0 \\ 0 & -3(1-t^2) & 0 & 0 \\ 0 & 0 & -3(1-t^2)\cos^2\lambda_0 & 0 \\ 0 & 0 & 0 & -3(1-t^2)\cos^2\lambda_0\cos^2\lambda_1 \end{pmatrix}$$

in the $(t, \lambda_{0,} \lambda_{1,} \lambda_2) - world$

Energy-impulse tensor

Diverse sources ([Fließbach], [Schröder], [Wiki]) propose

$$T^{\mu\nu} = \left(\rho + \frac{p}{c^2}\right) u^\mu u^\nu - p\, g^{\mu\nu}$$

with

$$u^\mu = \frac{\partial x^\mu}{\partial \tau}$$

In a local comoving reference system (Minkowski space-time) this yields a velocity

$$u^\mu = \begin{pmatrix} c \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ and } g^{\mu\nu} \rightarrow \eta^{\mu\nu} = diag(1,-1,-1,-1)$$

and an energy-impulse tensor

$$T^{\mu\nu} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

Meanings:

ρ mass density $\left[\frac{kg}{m^3}\right]$,

c vacuum speed of light $\left[\frac{m}{s}\right]$ and

p isotropic pressure $\left[\frac{N}{m^2}\right]$

for an ideal „cosmic“ fluid.

Following

$$T_{\mu\nu}=g_{\mu\rho}g_{\nu\sigma}T^{\rho\sigma}$$

we get the covariant version as needed in Einstein's field equations.

Einstein's field equations

Due to the changed sign of the curvature tensor the signs of the Ricci- and Einstein tensor change too. So the sign of the right hand side has to be changed too for maintaining the validity of the version of [Fließbach]:

$$R_{\mu\nu}-\frac{R}{2}g_{\mu\nu} \;=\; \frac{8\pi G}{c^4}T_{\mu\nu}$$

Let's write

$$R_{\mu\nu}-\frac{R}{2}g_{\mu\nu} \;=\; \kappa\, T_{\mu\nu}$$

with

$\kappa = coupling\ constant\left[\frac{s^2}{kg\, m}\right]$, also denoted as Einstein's gravitational constant,

to simplify the symbolic treatment a bit.

We should also think about the units of the involved parameters for once getting appropriate numerical values.

Let's have a closer look to the energy-impulse tensor:

$$T^{\mu\nu}=\left(\rho+\frac{p}{c^2}\right)u^\mu u^\nu-p\,g^{\mu\nu} \quad ,$$

describing the contents of the universe as an ideal „cosmic“ fluid.

c represents the maximum possible propagation velocity e.g. of light. c is a constant value within the hypersurface world, but a variable value with respect to the embedding hypersurface:

$$c=\frac{A}{\sqrt{1-t^2}}$$

It's the velocity $\frac{dx^0}{dt}$ of a non-moving observer or a particle along the „time“-axis x^0 .

$u^\mu=\frac{\partial x^\mu}{\partial t}$ refers to $\begin{pmatrix} x^0 \\ x^1 \end{pmatrix}=A\begin{pmatrix} \frac{\arcsin t}{\sqrt{1-t^2}} \\ \lambda(t) \end{pmatrix}$, ignoring x^2 and x^3 . We also set x^1 to zero because of regarding the particle as non-moving.

Now we solve $G_{00}=\kappa\, T_{00}$ for ρ :

$$\rho \;=\; \frac{(A^2\kappa\, p-3)(1-t^2)^2 - A^4\kappa\, p(1-t^2)}{A^6\kappa} \;=\; \left(\frac{p}{A^4}-\frac{3}{A^6\kappa}\right)(1-t^2)^2 - \frac{p}{A^2}(1-t^2)$$

and $G_{11}=\kappa\, T_{11}$ for p

$$p=\frac{3}{A^2\,\kappa}$$

Substituting this for p in the expression for ρ yields the corresponding mass density

$$\rho \;=\; -\frac{3(1-t^2)}{A^4\kappa}$$

For $t=0$ we might feel tempted to calculate a value of

$\rho' \approx -5{,}119\cdot 10^{-62}\left[\frac{kg}{m^3 s^2}\right]$ with $A \approx 13{,}7\cdot 10^9\, ly \approx 1{,}296\cdot 10^{26}\, m$

But taking care of choosing proper units, the unit of $(1-t^2)$ has to match the units of the other parameters. So when t is given in seconds the unitless 1 in $(1-t^2)$ has to correspond to the age of the universe in seconds squared.

Age of universe: $t_u \approx 13{,}7\cdot 10^9\, a \approx 4{,}323\cdot 10^{17}\, s$ and $1 \rightarrow (t_u)^2 \approx 1{,}869\cdot 10^{35}[s^2]$

Finally we get a well-known numerical value for the mass density

$$\rho \approx -9{,}569\cdot 10^{-27}\left[\frac{kg}{m^3}\right]$$

We have been assuming that the actual cosmic time is close to zero $t=0$, when the size of the universe reaches its maximum. This might hold true because of a vastly flat space-time which we encounter nowadays.

The pressure p remains fixed to

$$p \;=\; 8{,}6\cdot 10^{-10}\left[\frac{N}{m^2}\right]$$

and the relation

$$p=-\,\rho\, c^2$$

can be valid all-over due to the variable maximum speed when regarded from outside.

The absolute numerical value of ρ for $t=0$ just reflects the critical mass density required for a closed universe.

Discussion

How shall we interpret the results so far?

We have a pure geometrical model which yields a corresponding metric. We have no mass and no energy. But assuming that our chosen geometry is caused by matter and energy would require some negative mass density when the pressure is positive or maybe vice versa.

The negative mass density causes the universe to expand and the positive pressure holds it together. As both are more or less equal we always have an equilibrium state.

That allows for smoothly or maybe discretely changing hypersurface universes which are probably causally connected. The transition wouldn't need any extra energy.

And if real masses or real energy are added to such a universe?

Well, the equilibrium state gets disturbed in a sense that gravitational attraction becomes dominant.

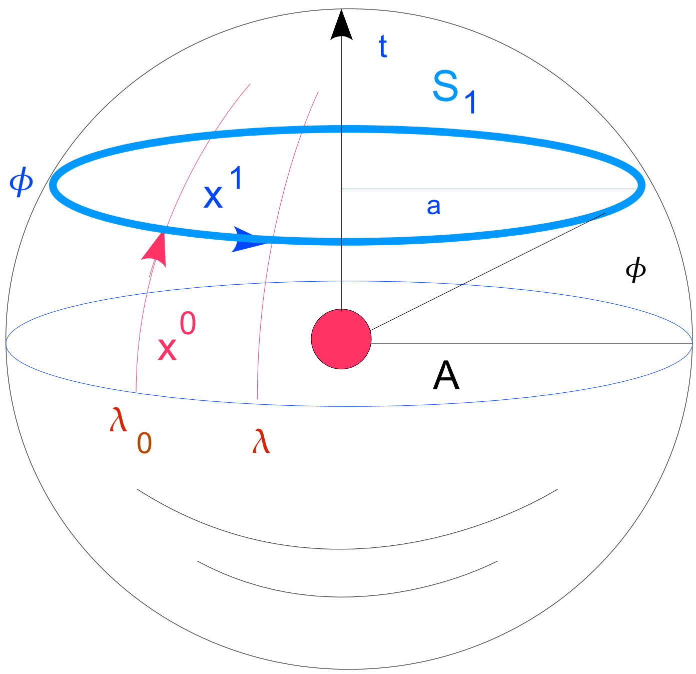
Changing the point of view

From Global to Local

Remember the path element in particular

$$(ds)^2 = g_{\mu\nu} dx^\mu dx^\nu = g_{00} dt\, dt + g_{11} d\lambda\, d\lambda = \frac{A^2}{1-t^2} dt^2 + A^2 (1-t^2) d\lambda^2$$

and let us now introduce a reference system $\{x_0, x_1\}$ pretending a flat space-time:



$$a(t) = A\sqrt{1-t^2}$$

$$x^0 = A\phi = A\arcsin(t)$$

$$x^1 = a \cdot (\lambda - \lambda_0)$$

Figure 2: Choosing a reference system with an apparently flat metric.

To achieve this we introduce the coordinates x_0 and x_1 obeying the metric

$$g_{\mu\nu}(x^0,x^1)=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

This yields of course (now contravariant and covariant coordinates are the same)

$dx_0^2=\frac{A^2}{1-t^2}dt^2$ and $dx_1^2=A^2(1-t^2)d\lambda^2$ or

$dx_0=\frac{A}{\sqrt{1-t^2}}dt=\frac{A^2}{a(t)}dt$ and $dx_1=A\sqrt{1-t^2}d\lambda=a(t)d\lambda$ respectively.

Integration leads directly to the values of the newly chosen coordinates

$x_0=A\arcsin t$ and $x_1=A\sqrt{1-t^2}\lambda=a(t)\lambda$, (we set $\lambda_0=0$ for the sake of simplicity)

Differentiation with respect to t results in

$\frac{dx_0}{dt}=\frac{A}{\sqrt{1-t^2}}=\frac{A^2}{a(t)}$ and

$$\frac{dx_1}{dt}=\frac{d\,a(t)}{dt}\lambda+a(t)\frac{d\lambda}{dt}$$

Using $\frac{d\,a(t)}{dt}=-\frac{At}{\sqrt{1-t^2}}=-\frac{A^2}{a(t)}t$ we may write

$\frac{dx_1}{dt}=-\frac{A^2}{a(t)}t\,\lambda+a(t)\frac{d\lambda}{dt}=\frac{A^2}{a(t)}\left(-t\,\lambda+(1-t^2)\frac{d\lambda}{dt}\right)$ to finally derive a velocity within the new reference system:

$$\frac{dx_1}{dx_0}=-t\lambda+(1-t^2)\frac{d\lambda}{dt}$$

This will get two distant observers ($\lambda>0$) to believe to be in mutual movement although they are not moving themselves ($\frac{d\lambda}{dt}=0$ and $t\neq 0$).

Trying to find the derivative of $\frac{dx_1}{dx_0}$ with respect to x_0 via $\frac{d}{dx_0}=\frac{d}{dt}\frac{dt}{dx_0}$ reveals

$\frac{d^2x_1}{dx_0^2}=\frac{dt}{dx_0}\left[-\lambda-3\mathrm{t}\frac{d\lambda}{dt}+(1-t^2)\frac{d^2\lambda}{dt^2}\right]$ an acceleration which will get two distant observers ($\lambda>0$) to believe to be attracted by an unknown force although they are not moving or accelerating themselves ($\frac{d\lambda}{dt}=0$, $\frac{d^2\lambda}{dt^2}=0$ and $-1<t<1$).

Local

The reference system as described by $\{x_0, x_1\}$ already comes close to the reference system which might be found from „inhabitants“ of the $S_1 - Universe$. But the coordinates of $\{x_0, x_1\}$ still have equal rights and do not need different units.

Indeed an observer in the $S_1 - Universe$ cannot see the x_0 coordinate. Only the x_1 coordinate is directly perceptible. But he/she may deduce some extra coordinate corresponding to x_0 because there are changes detectable within their universe.

So they might construct a reference system $\{\tau, \xi\}$ with τ corresponding to a parameter to be used to describe changes and ξ corresponding to a parameter to describe spatial distances. The first parameter will be called „time“ and the latter one „space“.

Postulates

Postulate 1

A (n-1)-dim hyperplane „glides“ evenly through the (n-1)-dim hypersurface of a n-dim hypersphere (radius A) und provides as intersections a series of (n-2)-dim hypersurfaces (with actual radii $a(t)$) serving as a dynamic „living room“ for (n-2)-dim „beings“.

$\phi = \arcsin(t)$ resp. $\sin \phi = t$, with t linearly increasing from -1 to $+1$ (cf. Figure 2)

Postulate 2

Each step in t -direction with an arbitrary small but constant step size causes a variable step in the direction of x_0 and allows a step in the x_1 -direction (or generally spoken in the spatial direction) of maximum the same size:

$\left|\frac{dx^1}{dx^0}\right|_{max} = 1$ which results in a maximum speed of

$v_{max} = \left|\frac{d\xi^1}{d\tau}\right|_{max} = c\left[\frac{m}{s}\right]$ in the $S_1 - Universe$ with suitably chosen units.

It is worth to mention that in a reference system with a maximum speed and a time coordinate which is not directly accessible (as in $\{\tau, \xi\}$) you have to deal with all the consequences of the theory of special relativity.

Discussing Equations

Velocity

Being observers in the $S_1\!-\!Universe$ and regarding the local velocity equation

$\frac{dx_1}{dx_0} = -t\lambda + (1-t^2)\frac{d\lambda}{dt}$ reveals an escape velocity when $t<0$ or an approach velocity when $t>0$ which depends on transmit time t and the distance λ at that time.

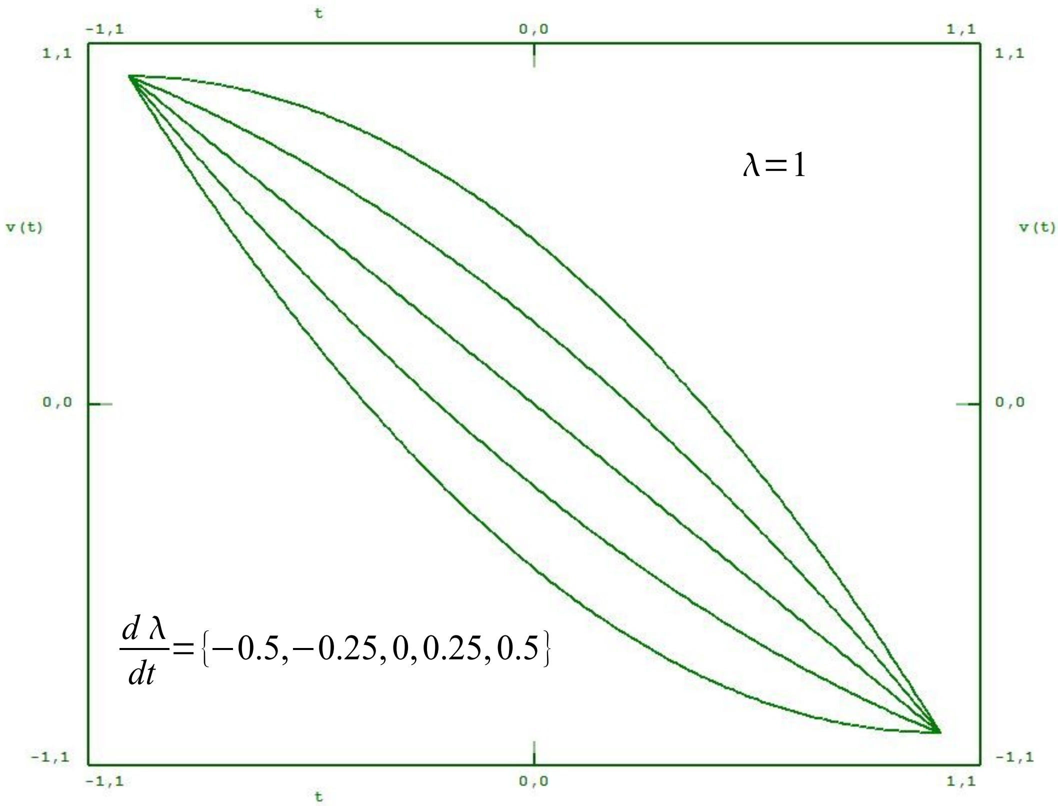


Figure 3: Some examples for the mutual velocity of two observers $\lambda=1$ apart for the specified proper initial velocities during the lifetime of their universe.

Inserting $t=\sin\frac{x_0}{A}$ and $\lambda=\frac{x_1}{A\sqrt{(1-t^2)}}=\frac{x_1}{A\cos\frac{x_0}{A}}$ yields for non-moving observers

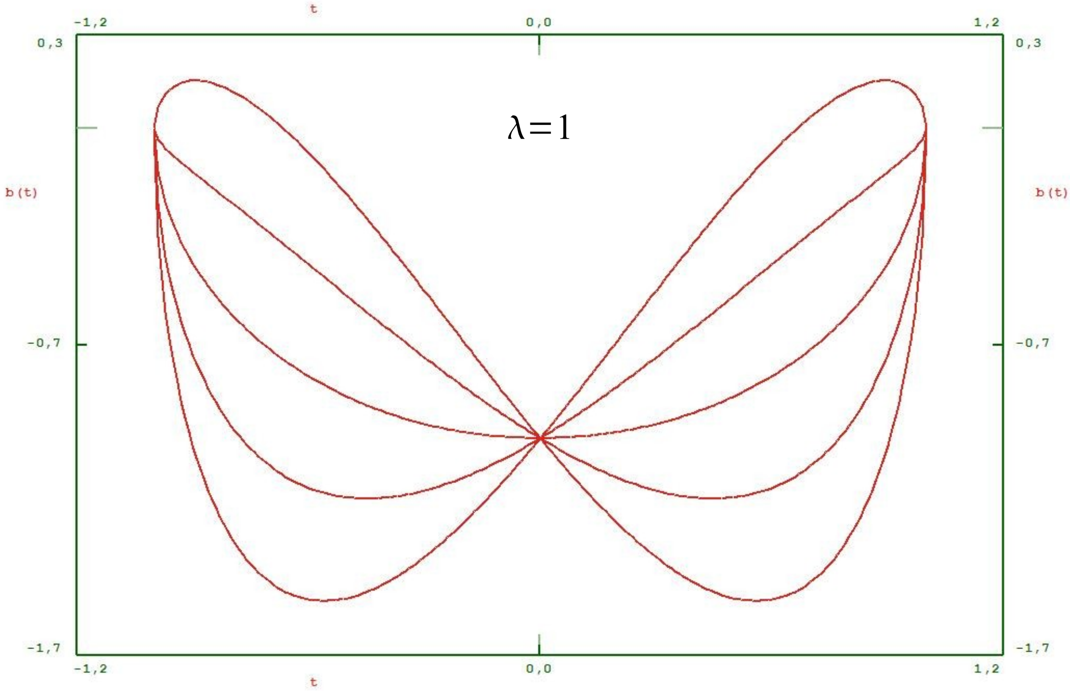
$\frac{dx_1}{dx_0} = -\sin\left(\frac{x_0}{A}\right)\frac{x_1}{A\cos\frac{x_0}{A}} = -\frac{1}{A}\tan\left(\frac{x_0}{A}\right)x_1$ an intrinsic variable speed as long as the distance x_1 does not vanish. In units of the $S_1\!-\!Universe$ the factor $\frac{1}{A}$ becomes

$\frac{c}{A} \simeq 2{,}313\cdot 10^{-18}\ \frac{1}{s} \simeq 71{,}4\frac{km}{s\,MPc}$ a value known as Hubble's constant.

Acceleration

The locally ($\{x_0, x_1\}$ -system) perceived acceleration

$\frac{d^2 x_1}{dx_0^2} = \frac{dt}{dx_0}\left[-\lambda - 3t\frac{d\lambda}{dt} + (1-t^2)\frac{d^2\lambda}{dt^2}\right]$ is even present when observers and/or objects are non-accelerated. Yes, even when they do not move with respect to each other.



$$\frac{d\lambda}{dt} = \{-0.5, -0.25, 0, 0.25, 0.5\}$$

Figure 4: Two observers $\lambda = 1$ apart might describe the acceleration ruling them like these nice curves.From left to right they correspond to the different angular speeds $\frac{d\lambda}{dt}$ listed above.

Although residing in a non-moving state ($\frac{d\lambda}{dt} = 0$, $\frac{d^2\lambda}{dt^2} = 0$) the observers experience an acceleration

$\frac{d^2 x_1}{dx_0^2} = \frac{dt}{dx_0}[-\lambda] = \frac{a(t)}{A^2}\left[-\frac{x_1}{a(t)}\right] = -\frac{x_1}{A^2}$.

In units of the $S_1 - Universe$ this reads as

$\frac{d^2\xi_1}{d\tau^2} = -\frac{c^2}{A^2}\xi_1$ with ξ_1 respresenting the spatial component aligned with x_1 and τ representing the time-like component tied numerically to $\tau \sim \frac{x_0}{c}$ via the constant maximum speed c . Inserting nowadays values for c and A yields

$\frac{d^2 \xi_1}{d \tau^2} \approx -5{,}35 \cdot 10^{-36} \ \xi_1 \left[\frac{m}{s^2}\right]$ a very tiny amount compared to about $10 \left[\frac{m}{s^2}\right]$ which we experience when living on the surface of the earth.

But let us now assume that this very tiny amount of acceleration is caused by some matter with mass M in the distance of the radius A of the universe.

We start with

$\frac{d^2 \xi_1}{d \tau^2} \approx -5{,}35 \cdot 10^{-36} \ \xi_1 \approx -\frac{GM}{\xi_1^2}$ and solve for

$M \approx 5{,}35 \cdot 10^{-36} \ \frac{\xi_1^3}{G}$ to recognize the cubic dependence of distance and to find the numerical value

$M \approx 5{,}35 \cdot 10^{-36} \ \frac{(1{,}296 \cdot 10^{26})^3}{6{,}67384\text{e-}11} kg \approx 1{,}75 \cdot 10^{53} kg$ indicating a tremendous big bunch of (virtual) matter acting gravitationally but being invisible. Indeed that amounts to several times the matter of stars, galaxies and intergalactic gas being detected so far.

Due to the cubic dependence of distance this will be barely recognizable in our solar system or nearby. But already in the distance of our milky ways' radius this represents a virtual mass of about 4 million suns.

Investigating Light Paths

Nearly all information about our universe reaches us via electromagnetic waves. So we shall have a closer look to their paths and the time they need to travel the vast distances to be found in the universe.

Although we are only regarding a pure geometrical universe without matter and energy so far we are going to postulate an information propagation service like light speeding with the utmost velocity $\left|\frac{dx_1}{dx_0}\right|_{max} = 1$ or $\left|\frac{d \xi_1}{d \tau}\right|_{max} = c$ respectively.

All we have to do is integrating the equations found so far.

Redshift

Light shows specific wavelengths which get distorted due to mutual velocities and the change of size of the whole universe. We consider

$z_c = \frac{\lambda_B}{\lambda_A} - 1 = \frac{a(t_B)}{a(t_A)} - 1$ the cosmological redshift and $z_D = \frac{\lambda_B}{\lambda_A} - 1 = \sqrt{\frac{1-v}{1+v}} - 1$ the Doppler-redshift due to mutual velocities when the signal was transmitted.

Measuring a redshift of $z_g = 1$ means in a cosmological sense that the radius of the actual world is twice as big as the radius of the former world when the signal was transmitted.

In the sense of mutual velocity (Doppler effect) the observer will assign an escape velocity of $\frac{3}{5}c$ to the signal source when treating himself as being stationary.

Redshift is so important because we really can measure it when comparing patterns of spectral lines of certain elements produced by distant sources to those emitted in our labs. And so it is essential to understand how redshift emerges

Another important value which we really can measure is the brightness of distant light sources. Distance is much harder to determine, but all three together provide the key for understanding the overall framework of the universe.

Integration variable

The independent variable for integration should change smoothly and evenly, so we shall use the reference system $\{t,\lambda\}$ as our target system and perform integration from t_A to t_B .

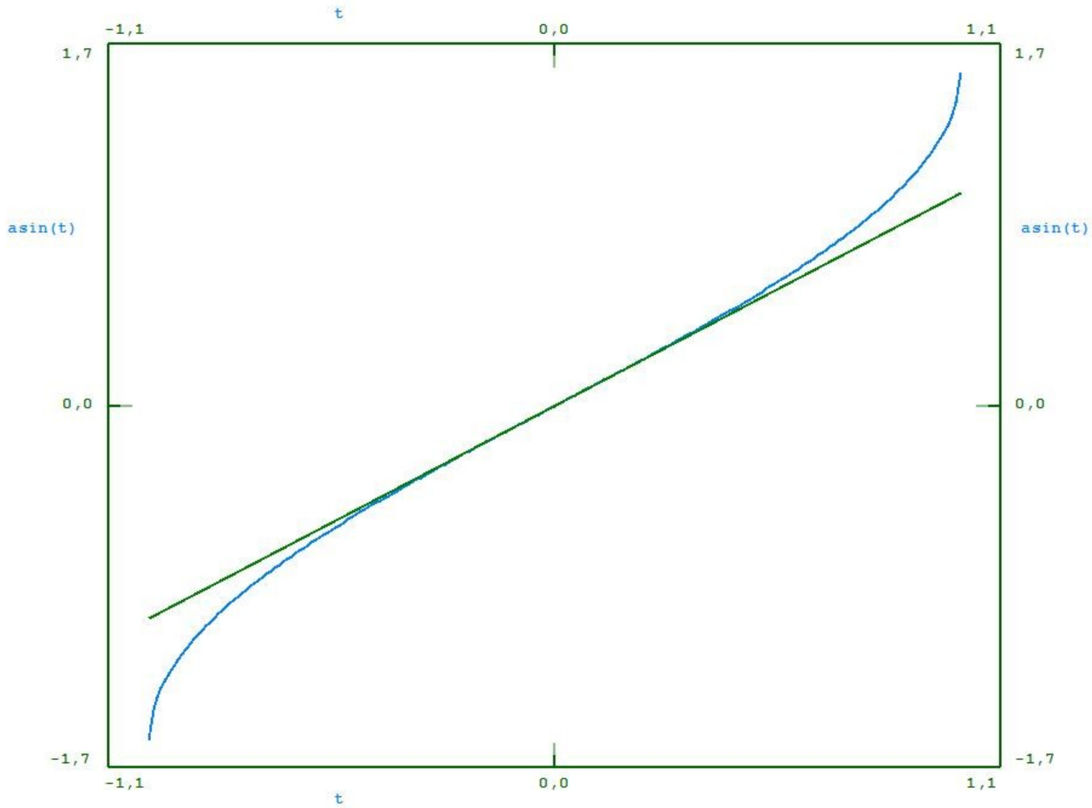


Figure 5: Linear measure for the even flow of cosmic time t in comparison to the non-linear local flow of time $x_0 = A\arcsin(t)$. About $t=0$ they look mostly the same.

The equations to be integrated

To get the values in the $\{x_0, x_1\}$ -system we integrate

$$\frac{dx_0}{dt} = \frac{A^2}{a(t)} \quad \text{with} \quad a(t) = A\sqrt{1-t^2}$$

and

$$\frac{dx_1}{dt} = \frac{dx_0}{dt}\left(-t\lambda + (1-t^2)\frac{d\lambda}{dt}\right)$$

numerically by taking into account that light propagates with constant maximum velocity

$\left|\frac{dx^1}{dx^0}\right|_{max} = 1$ in the $\{x_0, x_1\}$ -system, but with variable velocity in the $\{t, \lambda\}$ -system:

$$\frac{d\lambda}{dt} = \frac{\frac{dx^1}{dx^0} + \lambda t}{1-t^2} = \frac{\pm 1 + \lambda t}{1-t^2} \quad \text{as found from equating} \quad \left|\frac{dx_1}{dt}\right| = \frac{dx_0}{dt}$$

The angular distance λ has to be integrated simultaneously with x_0 and x_1 .

We already found $x_0 = A \arcsin t$ thus giving $\Delta x_0 = A(\arcsin t_B - \arcsin t_A)$ which may be used to control the integrated value.

The starting value for x_1 is $x_{1A} = a(t_A)\lambda_0$ with λ_0 the initial angular distance.

Travel time of signals

$$\lambda = 1$$

departure t_A	*arrival* t_B
-0,999	-0,9960
-0,99	-0,960
-0,9	-0,632
-0,8	-0,321
-0,7	-0,061
-0,6	0,156
-0,5	0,336

Figure 6: Time schedule for signals to bridge the angular distance of $\lambda = 1$

Light travelling from dawn to dusk

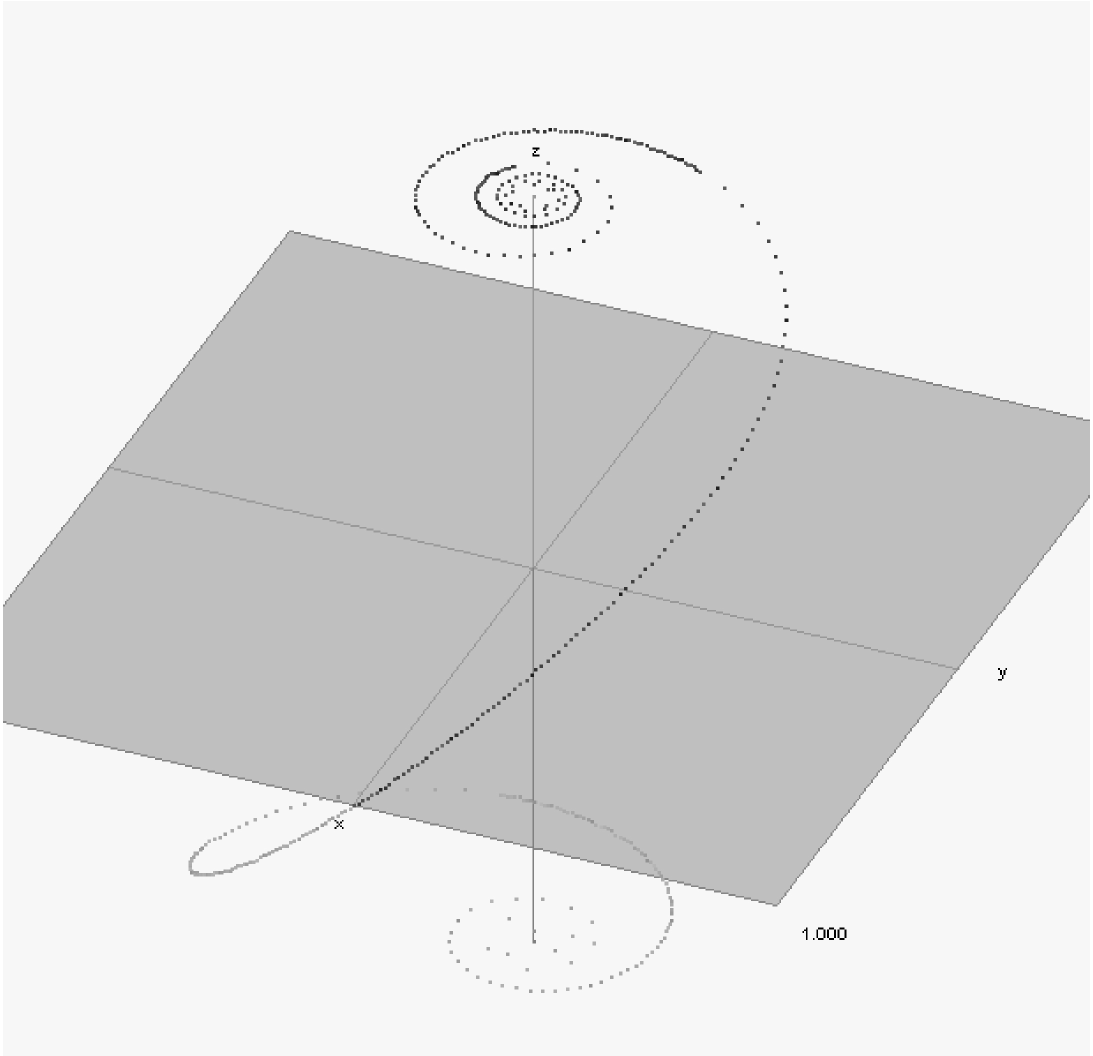


Figure 7: An example for a light path from the very beginning of the universe (bottom) to the distant future (top). The resolution had to be increased several times round the north pole of our universe (indicating the future). The pole regions show that light could easily traverse the whole universe not only once but many times.

Signals arriving nowadays

Via the straight path

t_A	x^0	$age[Gy]$	L	$\frac{\lambda}{2\pi}$	z_D	z_c	z_{ges}
-0,3760000	-0,385	10,338	0,559371	-0,066209	0,171	0,079	0,250
-0,4930000	-0,516	9,204	0,764562	-0,094307	0,351	0,149	0,500
-0,6100000	-0,656	7,978	1,005467	-0,131771	0,744	0,262	1,006
-0,6953000	-0,769	6,994	1,220056	-0,170253	1,609	0,391	2,000
-0,7248000	-0,811	6,629	1,306250	-0,187288	2,549	0,451	3,001
-0,7378000	-0,830	6,463	1,346835	-0,195650	3,528	0,481	4,009
-0,7444300	-0,840	6,377	1,368227	-0,200149	4,508	0,498	5,005
-0,7543200	-0,855	6,246	1,401079	-0,207182	9,478	0,523	10,001
-0,7580196	-0,860	6,197	1,413674	-0,209920	99,477	0,533	100,010

Figure 8: Light reaching us nowadays on the direct path was transmitted not earlier than about 6 billion years after the "big bang"

The columns of Figure 8 have the meanings:

t_A transmit time in the $\{t,\lambda\}$ -system

x^0 transmit time in the $\{x_0, x_1\}$ -system

$age[Gy]$ age of the universe in Giga-years when the signal was transmitted (based on assumed todays' age of 13.7 billion years)

L path length relative to the radius of the universe

$\frac{\lambda}{2\pi}$ angular path length relative to one revolution

z_D Doppler-redshift

z_c cosmological redshift

z_g measured redshift as sum of $z_D + z_c$

No signal transmitted more than 7.5 billion years ago may reach us via the direct path because of the more and more increasing redshift which lets fade away the electromagnetic wave into the realm of insignificance.

This looks like an unsurmountable hurdle in the presented theory giving evidence for being falsified. Because after all the microwave background which is clearly measurable dates back to about 400 thousand years past the „big bang“.

So, shall we speak about a bath of photons being expanded to nowadays wavelengths corresponding to a temperature of about 3 Kelvin ? If we can't assign sources to the light it might stream in stochastically without allowing us to detect structures in contradiction to the measurements taken by the satellites COBE and WMAP.

Via the indirect path

Figure 7 already indicates that light may arrive nowadays emanating from our ancient cosmic neighbourhood at ancient times. Due to much longer routes the signals got rather weak. The redshift is nearly completely attributed to cosmic expansion. Interestingly the signals have traversed the whole universe many times thus having supported a very busy exchange of information in the early universe.

t_A	x^0	$age[Gy]$	L	$\frac{\lambda}{2\pi}$	z_D	z_c	z_{ges}
-0,97651698	-1,354	1,894	3,268389	-1,000001	0,0	3,64	3,64
-0,99327642	-1,455	1,012	4,250381	-2,000001	0,0	7,64	7,64
-0,99686423	-1,492	0,691	4,849294	-3,000001	0,0	11,64	11,64
-0,99819194	-1,511	0,525	5,281682	-4,000001	0,0	15,64	15,64
-0,99882527	-1,522	0,423	5,620323	-5,000001	0,0	19,64	19,64
-0,99969717	-1,546	0,215	6,684971	-10,000001	0,0	39,64	39,64
-0,99999688	-1,568	0,022	10,279652	-100,000001	0,0	399,64	399,64
-0,99999977	-1,570	0,006	12,311592	-365,000001	0,0	1459,64	1459,64
-0,99999997	-1,571	0,002	13,894297	-1000,000001	0,0	3999,64	3999,64

Figure 9: Light having already traversed the whole ancient universe at least once may arrive today too. The meaning of the columns is the same as in figure 8.

Now the situation looks much more relaxed because ancient light transmitted before the age of 6 billion years may reach us as well. Light with a redshift of about 1500 may quite possibly bear witness to the time when the universe got transparent for electromagnetic waves.

All the information sent from the ancient cosmic neighbourhood might arrive nowadays as long as the Doppler-redshift does not let it fade away. In figure 9 the initial distance $\frac{\lambda_0}{2\pi}$ was chosen to be a millionth of one complete revolution. That's why the Doppler-redshift z_D doesn't matter. But of course the initial distance may be chosen a bit bigger to produce a bigger redshift which will get added to the cosmological redshift z_c .
The minimum redshift to be measured for indirect signals is 3.64 when having already performed one revolution about the universe. For two revolutions the minimum redshift amounts to 7.64 and so on. Figure 10 shows the connection between the initial distance depicted as revolution („Umrundung“) and the allowed time span within the signal had to be transmitted.

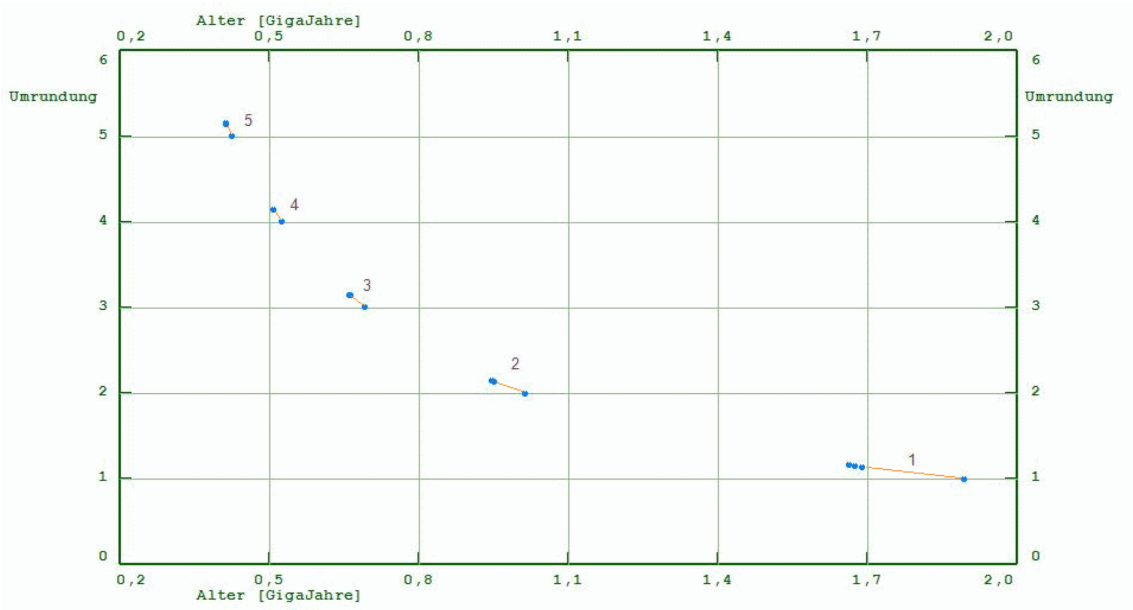


Figure 10: Signals coming to us via the indirect path can only bring information emanating from distinct time intervals.

Now let's have a closer look to very early indirect signals having propagated already 360 to 370 times round the universe (see figure 11).

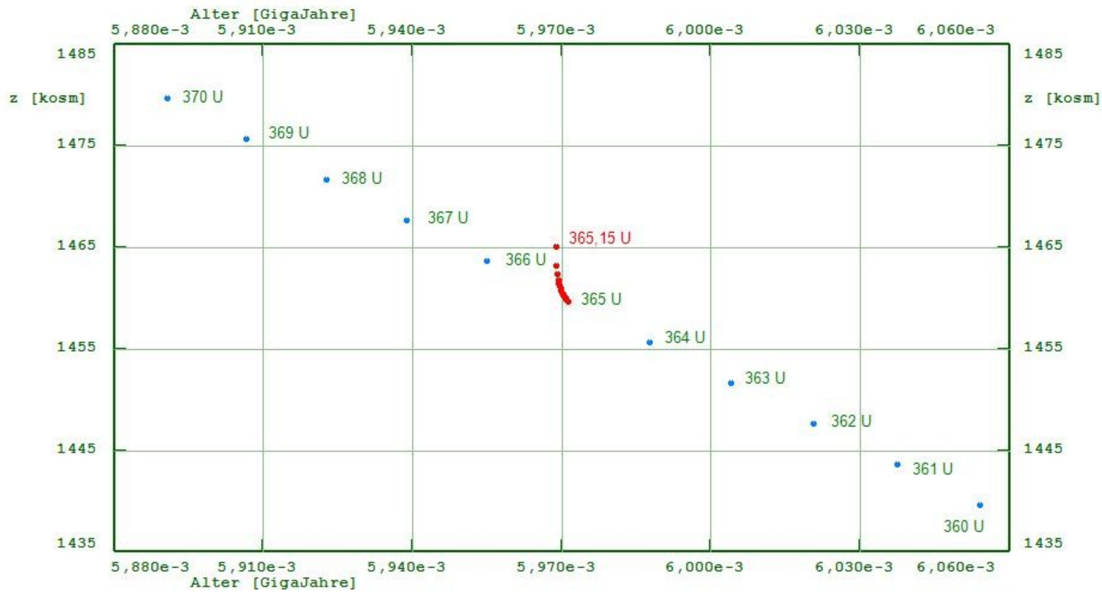


Figure 11: Redshift versus age of universe for revolutions 360 to 370. The red tail of rev. 365 indicates the span of additional redshift when the initial distance is increased a bit and should be added to the other points too.

The redshift of about 1435 to 1485 can be ascribed to the microwave background showing photons having already traversed the whole universe about 365 times. Their original wavelength got lengthened by the redshift factor (cosmic expansion) and occurs nowadays as corresponding to a black body radiation of a temperature of about 3 Kelvin.

Stretching of time

There is not only the effect of redshift which gets wavelengths stretched but also an effect responsible for stretching time intervals which need to be considered when investigating light paths and series of events.

The first one is well known but the second one is not. When investigating supernovae of type Ia which should show the same course of brightness (due to the same physical processes which made them explode) cosmologists recognized that the course of brightness depends on distance. The farther away the longer the duration. When matching the different courses of brightness empirically they could deduce distances of the sources which led them directly to the strange result that the universe is expanding in an accelerated manner.

We first look at the effect of „time-stretching“ for direct signals as described in figure 8. They all arrive here at $t=0$ being transmitted at times x_0 as depicted in the second column. Now we calculate the corresponding arrival times when the signals get sent a small amount $\Delta x_0=0{,}001$ of time later. The cosmic time dilation follows simply as

$$d_c=\frac{\Delta x_{0R}}{\Delta x_{0T}}$$

the quotient of the difference of receive times divided by the difference of transmit times (cf. Figure 12).

$[Gy]$	L	$\frac{\lambda}{2\pi}$	d_c	z_D	z_c	z_{ges}
10,338	0,559371	-0,066209	1,156	0,171	0,079	0,250
9,204	0,764562	-0,094307	1,292	0,351	0,149	0,500
7,978	1,005467	-0,131771	1,505	0,744	0,262	1,006
6,994	1,220056	-0,170253	1,743	1,609	0,391	2,000
6,629	1,306250	-0,187288	1,853	2,549	0,451	3,001
6,463	1,346835	-0,195650	1,907	3,528	0,481	4,009
6,377	1,368227	-0,200149	1,936	4,508	0,498	5,005
6,246	1,401079	-0,207182	1,982	9,478	0,523	10,001
6,197	1,413674	-0,209920	1,999	99,477	0,533	100,010

Figure 12: Transmitting signals a small amount of time later than shown in figure 8 helps to reveal an effect we might call cosmic time-stretching or cosmic time dilation. Distant series of events thus appear in slow motion with a moderate factor between 1 and 2.

So it is like watching ancient events in slow motion when distanced far away. The effect is quite moderate and does not exceed a factor of two. If something in the far past (when the universe was 6.2 Giga-years old) took one month time, so we will think it lasted about two months.

We may state empirically that d_c varies linearly like

$$d_c \approx 1+1{,}88\cdot z_c$$

depending on the cosmological redshift z_c .

Cosmic time dilation gets really big when considering signals following an indirect path (cf. Figure 13). Very distant events appear at least in sevenfold slow motion and redshifted by a factor of about 3.64 .

$[Gy]$	L	$\frac{\lambda}{2\pi}$	d_c	z_D	z_c	z_{ges}
1,894	3,268389	-1,000001	7,13	0,0	3,64	3,64
1,012	4,250381	-2,000001	13,48	0,0	7,64	7,64
0,691	4,849294	-3,000001	19,79	0,0	11,64	11,64
0,525	5,281682	-4,000001	26,09	0,0	15,64	15,64
0,423	5,620323	-5,000001	32,38	0,0	19,64	19,64
0,215	6,684971	-10,000001	63,81	0,0	39,64	39,64
0,022	10,279652	-100,000001	629,32	0,0	399,64	399,64
0,006	12,311592	-365,000001	2294,36	0,0	1459,64	1459,64
0,002	13,894297	-1000,000001	6284,19	0,0	3999,64	3999,64

Figure 13: Cosmic time dilation is a real big thing when considering indirect signals. Distant events appear at least in sevenfold slow motion and show redshifts of at least 3.64

Looking at the signals which might describe the microwave background ($\frac{\lambda}{2\pi}\approx 365$) opens a new era of cosmic slow motion. The factor is now about 2300, so a short event then of maybe 1 day appears nowadays as long-winded tale lasting more than 6 years. Interestingly the universe was already 6 million years old then (cf.. 1st column in figure 13)

For indirect signals we might state empirically that d_c varies linearly like

$d_c \approx 7{,}13 + 1{,}57 \cdot (z_c - 3{,}64)$

depending on the cosmological redshift z_c .

So, whenever a time dilation factor of more than 7 should be observed one day but none between 2 and 7, then the presented theory might gain a lot of credibility.

Very early signals

According to Wikipedia (and NASA maintaining the Hubble space telescope) the oldest signals come out of a time of about 600 million years past the „big bang“ and show redshifts up to $z_g = 8{,}55$ (Hubble Ultra Deep Field, cf. Figure 14)

If we ascribe this redshift to the cosmological expansion of the universe, the size then was only about 10.5% the size of nowadays. The corresponding time of transmission, supposing $t_B = 0$, amounts to $t_A = -0{,}9945$ or $x_A^0 = -1{,}466$ being equivalent to 6.68% of the age nowadays (about 915 million years of 13.7 billion years). According to Doppler the redshift of $z_g = 8{,}55$ means that the galaxy recedes with 97.8% of the speed of light. The distance then was about $\lambda_A \approx 0{,}9837$ corresponding ($t_B = 0$) to $13{,}477$ billion light years nowadays. The galaxy only had 223,3 million years time to develop.

Figure 14: A tiny region of the northern sky near the Big Dipper was photographed during several days by the Huble Space Telescope revealing never before seen galaxies extremely far away.(cf. NASA pictures of the day).

We might explain the redshift alternatively using figure 8 giving

$z_{ges}=8{,}53$, $t_A=-0{,}753$, $x_A^0=-0{,}853$ and $\lambda_A=-1{,}296$ corresponding to an age of the universe of 45.7% of nowadays' age (about 6,26 billion years, so giving the galaxy plenty of time to develop).

Or we may consult figure 9 to allow for indirect signals too and find in comparison to figure 8:

$[Gy]$	L	$\frac{\lambda}{2\pi}$	d_c	z_D	z_c	z_{ges}
6,377	1,368	-0,200	1,936	4,508	0,498	5,005
6,246	1,401	-0,207	1,982	9,478	0,523	10,001
1,012	4,250	-2	13,48	0,0	7,64	7,64
0,691	4,849	-3	19,79	0,0	11,64	11,64

Figure 15: Alternatives for explaining a redshift of 8.5. Direct signals (top) and indirect signals (bottom) differ in brightness (due to longer L) and tremendously in cosmic time dilation.

Figure 15 gives clear evidence that redshift is no longer reflecting distance unambiguously.

About Brightness

Before we can understand how a closed universe may fake an accelerated expanding one, we have to examine how brightness of astronomical objects is determined. That is because brightness is treated as a key for opening distances. If all objects had the same brightness this might indeed come true in a flat universe with a smooth and even flow of time.

Magnitudes

The ancient greek astronomers Hipparcos and Ptolemy divided the visible brightness of stars into 6 magnitudes. The brightest stars ($m=1$) should be a factor of 100 brighter than the faintest stars ($m=6$).

So we get

$$\Delta m = 2{,}5 \cdot \log \frac{I_1}{I_2}$$

a logarithmic ratio of intensities.

This means for one unit of magnitude

$$\Delta m = 1 \;\rightarrow\; \frac{I_1}{I_2} = 10^{0{,}4} \approx 2{,}512$$

and for twofold brighter sources of light

$$\frac{I_1}{I_2} = 2 \;\rightarrow\; \Delta m \approx 0{,}753$$

Counting photons

To simplify matters we try to determine brightness by counting photons which carry the energy. We are not really interested in the absolute value but in the ratio of photons received from different sources.

We count n_A photons received from an evenly emitting source of light L_A during a certain interval of time Δx_0 . Having sources emitting the same amount of energy in a flat 3-dimensional space with a linear flow of time the count n_A would only vary due to the distance squared.

We observe another source of light L_B and count n_B photons during the same time interval Δx_0 .

To compare L_B to L_A we have to correct n_B for

$n' = n_B \cdot \frac{d_{cB}}{d_{cA}}$ the cosmic time dilation or stretching of time,

$n''=n'\cdot\left(\frac{x_{1B}}{x_{1A}}\right)^2$ the distance and

$n'''=n''\cdot\frac{z_{gB}+1}{z_{gA}+1}$ the measured redshift

Loss of brightness

We regard a reference light source in about 3.4 Giga-lightyears distance and calculate the loss of brightness of equal but more distant light sources as being „measured“ in the postulated model universe.

$\xi\ [Gly]$	n_A/n_B	Δm_1	z_g	$\frac{z_{gB}+1}{z_{gA}+1}$	Δm_2	$\Delta m_1+\Delta m_2$
3,362	1,000	0,000	0,250	1,000	0,000	0,000
4,496	1,999	0,752	0,500	1,200	0,198	0,950
5,722	3,772	1,441	1,006	1,600	0,510	1,951
6,706	6,000	1,945	2,000	2,400	0,951	2,896
7,071	7,092	2,127	3,001	3,200	1,263	3,390
7,237	7,646	2,209	4,009	4,000	1,505	3,714
7,323	7,946	2,250	5,005	4,800	1,703	3,953

Figure 16: We observe equal light sources in varying distances to find out how big the loss of brightness would be when correcting for different effects.

The columns of figure 16 contain

$\xi\ [Gly]$ the distance in Giga-lightyears

n_A/n_B the ratio of photon counts corrected for distance and time-stretching

Δm_1 the corresponding difference in magnitudes

z_g the measured redshift

$\frac{z_{gB}+1}{z_{gA}+1}$ the ratio of measured redshifts

Δm_2 the corresponding difference in magnitudes

$\Delta m_1+\Delta m_2$ the final loss of brightness of more distant sources of light

In the supernova cosmology project only Δm_1 is being considered.

The values of figure 16 (plus the correction for distance only and time stretching only) are put into a diagram for getting a better overview (cf. Figure 17).

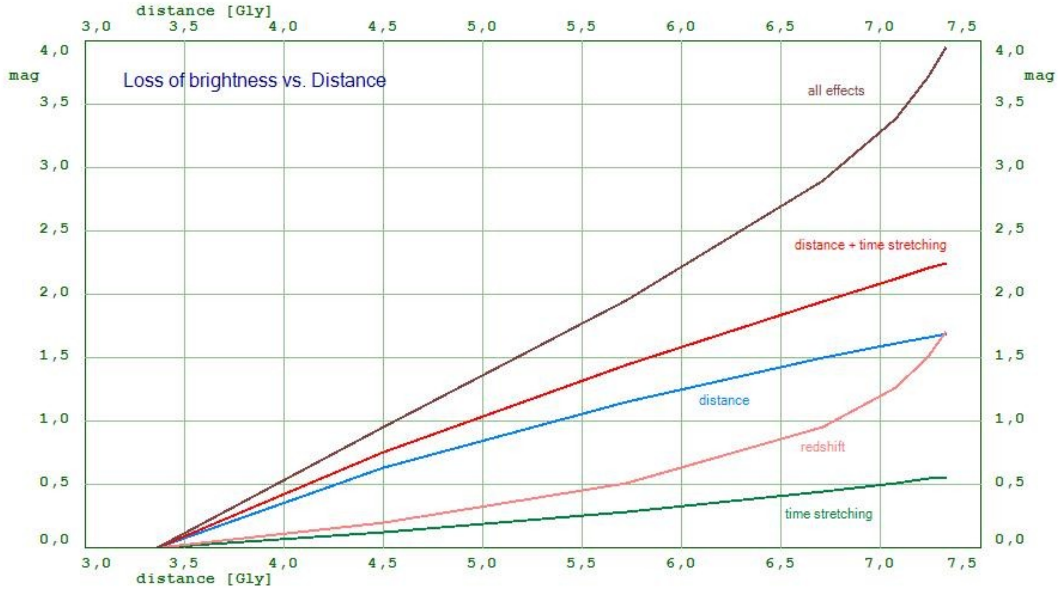


Figure 17: Loss of brightness versus distance when several corrections get applied.

Blue curve --> distance only, green curve --> time-stretching only, red curve --> distance+time-stretching, light red curve --> redshift only, brown curve --> all effects taken into account.

Omitting the correction Δm_2 leaves a gap which can only be filled by assuming a somewhat increased distance. Having a closer look at a light source 4 billion light years apart, the additional loss of brightness might be explained by an increased distance amounting to 4.17 billion light years (cf. Figure 18).

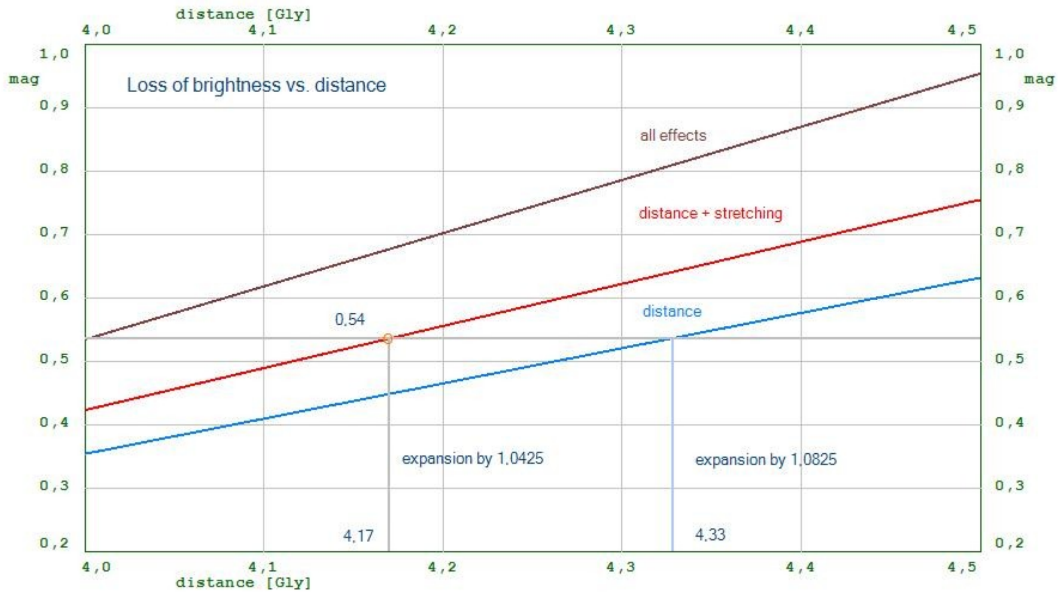


Figure 18: To explain the additional loss of brightness as depicted by the red and brown curves requires to assume a bigger distance if only the red curve is taken to correct the measured brightness.The diagram reflects the situation for a source of light in a distance of ***four*** *Giga-light years.*

Now, looking at a source of light 5 billion light years apart reveals a real interesting result (cf. Figure 19)

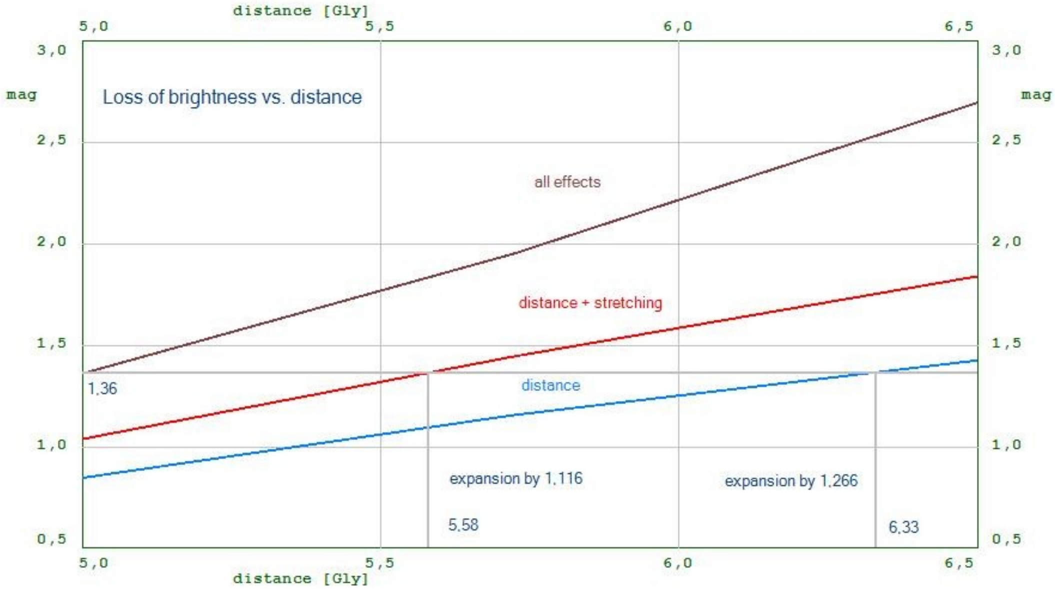


Figure 19: To explain the additional loss of brightness as depicted by the red and brown curves requires to assume a bigger distance if only the red curve is taken to correct the measured brightness.The diagram reflects the situation for a source of light in a distance of ***five*** *Giga-light years.*

We easily see that the required increase of distance is much bigger now than for a source of light „only“ 4 Gly away.
So for explaining the actual loss of brightness we now have to demand an accelerated expanding universe.

Accelerated Expansion

We look at figures 18 and 19 to summarize the results.

To explain the loss of brightness with only distance and stretching of time requires to explain the residual difference by an increased distance.

At a distance of 4 Gly a factor of 1,0425 (= 4,17 / 4,00) is needed
At a distance of 5 Gly a factor of 1,1160 (= 5,58 / 5,00) is needed

The expansion factor itself has to be increased by a factor of 1,0705 when distance moves from 4 to 5 Gly.

For the distance of the „big bang“ we might extrapolate a factor of $1{,}0705^{9{,}7} \cdot 1{,}0425 \rightarrow 2{,}02$ meaning that the source of the signal has to be twice as far away.

That's why cosmologists ask for an accelerated expansion of the universe. But this requires some negative mass or „dark energy“ as it is called nowadays.

Estimating the amount of „dark energy“

The simple equation

$$d=\frac{1}{2}\frac{GM}{r_u^2}t_u^2$$

used for calculating the distance traversed in a gravitational field shall help us to find an estimation for the quantity of „negative mass“ $-M_{de}$ necessary to drive away a bit of mass originally located at the edge of the universe about two times the radius r_u of the universe during its lifetime t_u so far (G means the gravitational constant of Newton).

A (positive) mass $M=1,75\cdot 10^{53}[kg]$ as found in chapters „*Characteristic Parameters*“ and „*Acceleration*“ is able to stop expansion and begin contraction of our universe (as only part of it is visible, the rest had to be demanded as some kind of „dark matter“ when presuming a flat universe).

A small test mass at the edge of the universe would fall about half the radius r_u of the universe towards the center during the time t_u of the age of the universe.

To achieve that the test mass does not fall in but does drift away twice as distant requires some repulsive force about three times stronger than the attractive force.

This can be concluded due to some simple-minded reasoning:

An additional (negative) mass $-M$ could just keep the test mass in the same distance. The twofold amount $-2M$ could move it away half the radius r_u and finally the threefold amount $-3M$ could repel it to twice the radius of the universe.

Some fictitious „negative mass“ called „**dark energy**“ does the job. No one knows what it really is. And it only comes into effect when looking at cosmic distances (!).

The actual composition of our universe must be stated as including

about 25% attractive matter (about 1/6th is visible, the rest goes into „dark matter“)
and
about 75% repulsive matter (nothing is visible, all goes into „dark energy“)

when trying to maintain a flat universe.

Conclusion

Today's brightness measurements of distant supernovae of type Ia suggest an accelerated expansion of the universe. And the stability of galaxies and clusters of galaxies requires more mass than is visible. The only explanation so far given for a flat space-time is some kind of „dark energy“ driving the expansion and some kind of „dark matter“ holding things together.

Suggesting a dynamically warped space-time which makes the universe expand at a decelerating rate, reach a maximum size and then contract at an accelerating rate, helps to overcome the concepts of „dark matter“ and „dark energy“.

But you have to break the rule that warping space-time may only be caused by matter and energy and accept that warped space-time might be an inherent feature of the universe itself.

Thanks

Thanks to God and all who helped me investigating some very interesting aspects of nature.

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https://play.google.com/store/apps/details?id=de.dieteregger.symbolic
Example scripts including the metric calculations may be found at
http://www.dieteregger.de/symbolic/scripts.html.en

Wikipedia-Links
http://de.wikipedia.org/wiki/Dichteparameter
http://de.wikipedia.org/wiki/Einsteinsche_Feldgleichungen
http://de.wikipedia.org/wiki/Energie-Impuls-Tensor
http://en.wikipedia.org/wiki/Hubble_Ultra-Deep_Field
http://de.wikipedia.org/wiki/Kugelkoordinaten
http://de.wikipedia.org/wiki/Riemannsche_Mannigfaltigkeit
http://de.wikipedia.org/wiki/Rotverschiebung
http://de.wikipedia.org/wiki/Supernova_Cosmology_Project
http://de.wikipedia.org/wiki/Universum

Appendix

Reduce

Reduce is Open Software and is available for Windows or Linux at

http://reduce-algebra.sourceforge.net/

and also free of charge as Android App named „Symbolic“
https://play.google.com/store/apps/details?id=de.dieteregger.symbolic
at the Google Play Store

It helped evaluating and / or checking several formulae like

Christoffel symbols:

$$\Gamma^{\kappa}_{\lambda\mu}=\frac{1}{2}g^{\kappa\nu}\left(g_{\mu\nu,\lambda}+g_{\lambda\nu,\mu}-g_{\mu\lambda,\nu}\right)$$

Riemannian curvature tensor:

$$R^{m}_{ikp}=\partial_p\Gamma^{m}_{ik}-\partial_k\Gamma^{m}_{ip}+\Gamma^{r}_{ik}\Gamma^{m}_{rp}-\Gamma^{r}_{ip}\Gamma^{m}_{rk}$$

Ricci tensor:

$$R_{ij}=R^{m}_{ijm}$$

Energy-impulse tensor:

$$T^{\mu\nu}=\left(\rho+\frac{p}{c^2}\right)u^{\mu}u^{\nu}-p\,g^{\mu\nu}$$

and the same in covariant form

$$T_{\mu\nu}=g_{\mu\rho}g_{\nu\sigma}T^{\rho\sigma}$$

Einstein's field equations:

$$R_{\mu\nu}-\frac{R}{2}g_{\mu\nu}\ =\ \frac{8\pi G}{c^4}T_{\mu\nu}$$

resp.

$$G_{\mu\nu}\ =\ \kappa\ T_{\mu\nu}$$

Scripts for calculating

Example scripts including the metric calculations may be found at
http://www.dieteregger.de/symbolic/scripts.html.en