## Appendix A

## Elements of Bond Graph Modelling

## A. 1 Definition

Bond graph modelling is based on considering the exchange of power between the elements of a system. A bond graph is a collection of multiport element bonded together. In the general sense it is a linear graph whose nodes are multiport element and whose branches are bond. The key of bond graph modelling is the representation (by a bond) of power as the product of efforts and flows with elements acting between these variables and junction structures to put the system together.

The product of effort and flow is then a power, as well known in electricity, where power is voltage times current. This is used in most disciplines, like hydraulics, where power is pressure times volume flow and mechanics where power is force times effort or torque times rotation frequency.

In thermodynamics, power, that is heat flow is also temperature times entropy current. This is much less known but very important and that is why we put it at the beginning of thermodynamics as Carnot equation 1.3.

In some bond graphs the product effort-flow does not have the physical dimension of a power. There are called pseudo bond graphs and need to be used in chemistry, thermodynamics and thermal engineering.

An important example of pseudo bond graphs is thermal and thermodynamics engineering. Here we have both the true Bond graph, with temperature and entropy current, whose product is a power, and the pseudo bond graph with temperature as effort and heat flow as flow. So in this latter formulation effort times flow is not a power and has no physical meaning, but is used for the practical reasons as explicating above in the fourth chapter.

The construction of a bond graph model of a given system is realised in three steps:

1. The first stage is to split up a system into a subsystems or the coupling of different selected phenomena in function of imposed hypothesis. The interconnection is realized by the pair effort and flow. One can say this the technological level of model building or word bond graph.
2. The second step is to reproduce the physical effects, including simplifying hypothesis. Here the interdisciplinary nature of bond graphs is very useful, and shows the exchange of power in a system. It also shows the storing, the transmission and dissipation of energy. Fur-
ther, the mathematical order and the type of dynamics are directly seen on the bond graph. Finally, the bond graph model obtained on a paper or computer screen can modify by adding more elements, like frictions, and inertia's compressibility's. This is the physical level of model building and analysis.
3. The third step is to write the constitutive equations of the elements or components according the selected causalities. This is the mathematical and algorithmic representation.

## A. 2 Bond Graph Elements with one Port

## A.2.1 Representation

The bond graph elements can be classified by their number of ports, from one to three, whilst multiport appear in the next section.

In each closed system, there is both conservation of energy and a continuous flow of power. The exchanged power between two connected systems A and B is represented by a power bond as shown on the figure A.1.


FIGURE A.1. Representation of bond graph one port element
From this comes the term bond graph. The sense of the half arrow gives the direction of the power. If the power flux is sometimes backwards, it is described by negative power.

## A.2.2 Used Variables in Bond Graph Modelling

The used variables in bond graph can be classified as follows:

1. Power or liaison variables. Here the exchanged power equals effort $e$ times flow $f$, which are in general function of time:

$$
P=e f
$$

2 Energy variables. These are defined by the following integrals equations:
$p(t)$ the momentum or impulse of a movement

$$
\begin{equation*}
p(t)=\int_{0}^{t} e(\tau) d \tau, p(0)=0 \tag{A.1}
\end{equation*}
$$

and $q(t)$ the general displacement

$$
\begin{equation*}
q(t)=\int_{0}^{t} f(\tau) d \tau, \quad q(0)=0 \tag{A.2}
\end{equation*}
$$

The following table A. 1 gives the meaning of the variables in the different domains.

| Domain | Effort | Flow | Momentum | Displacement |
| :---: | :---: | :---: | :---: | :---: |
|  | [Unit] | [Unit] | [Unit] | [Unit] |
|  | $e(t)$ | $f(t)$ | $p(t)$ | $q(t)$ |
| Electric | Tension | Current | magnetic flow | Electric charge |
|  | [Volt] | [Ampere] | [Weber] | [Coulomb] |
| Mechanics of rotation | Torque | Frequency | Angular momentum | Angle |
|  | [N.m] | [Rad/s] | N.s] | [Rad] |
| Mechanics of translation | Force | Velocity | Momentum | Volume |
|  | [ $N$ ] | [ $\mathrm{m} / \mathrm{s}$ ] | [ $N . s$ ] | $\left[m^{3}\right]$ |
| Hydraulics | Pressure | Volume flow | Momentum of | Volume |
|  | [Pa] | $\left[\mathrm{m}^{3} / \mathrm{s}\right]$ | pressure [Pa.s] | $\left[m^{3}\right]$ |
| Thermal | Tempe- $\text { -rature }[K]$ | Heat flow $[\mathrm{J} / \mathrm{s}]$ | - | Heat energy [J] |
| Thermodynamics | Tempe--rature $[K]$ | Entropy flow $[J /(K . S)]$ | - | Entropy [ $J / K$ ] |
| Chemistry | Chemical tension | Molar flow | - | molar mass |
|  | [J/mole] | [Mole/s] |  | [Mole] |
| Economy | Unit price of product | Flow of product | Economic momentum | Inventory |

TABLE A.1. Meaning of power and energy variables

Describing now the different elements according to their number of ports.

## A.2. 3 Passive Elements

These elements store or dissipate energy. The half arrow is usually entering the element. These elements are designated by $R, C$ and $I$.

- Resistance $R$. This is the generalized friction and connects effort and flow. It gives the dissipation of power (see below) and is found in all disciplines: electric resistors, hydraulic resistors, linear or not, mechanical friction, thermal friction in pseudo bond graphs. The law is of the form:

$$
\begin{equation*}
\Phi_{R}(e, f)=0 \tag{A.3}
\end{equation*}
$$

Examples are the electric law of Ohm, $U=R i$ or the hydraulic law of Bernoulli $P=K_{D} \dot{V}^{2}$ or Fouriers thermal law $\Delta T=K_{C} \dot{Q}$.

The graphical representations are given by figure A.2.


FIGURE 6.2. Graphical representation of $R$ element

- Capacitance C. It connects all effects connecting effort and displacement: spring, condenser, accumulator, storage reservoir and compressibility. So we have here storage of energy and the equation is

$$
\begin{equation*}
\Phi_{C}(e, q)=0 \quad \text { or } \quad \Phi_{C}\left(e, \int f(\tau) d \tau\right)=0 \tag{A.4}
\end{equation*}
$$

Examples are $u=\frac{1}{C} \int i d t$ or $u=\frac{q}{C}$ in electricity, $P=\frac{V}{C} \quad$ or $P=$ $\frac{1}{C} \int \dot{V} d t$ in hydraulics and $T=\frac{1}{C} \int \dot{Q} d t$ in thermic.

- Inertia I. This element is used for mass action, moment of inertia in rotary mechanics, inductance and inertance of a liquid. Its equation is:

$$
\begin{equation*}
\Phi_{I}(p, f)=0 \text { or } \Phi_{I}\left(f, \int e(\tau) d \tau\right)=0 \tag{A.5}
\end{equation*}
$$

Example: $u=L \frac{d i}{d t}$ or $\phi=L i$ in electricity and $P=\frac{\rho l}{A} \dot{V}$ in hydraulics. $l, A$ and $\rho$ represent respectively the length and the cross area of the pipe and the density of the fluid circulating in the pipe.

Both $C$ and $I$ elements are associated to storing energy. But in opposite to $C$ element, In the $I$ element, the energy is not conserved when the flow variable disappears: the energy is stored because of dynamic of the system.

## A.2.4 Active Elements

These elements supply power to the system. Naturally, this power must come from somewhere, but this is outside our range of interest. One distinguishes:

- Effort source (Se), imposing an effort, which can be function of time but independent of required flow. Examples is the electric voltage generator, pressure, force generator or simply weight force generator.
- Flow source (Sf), imposing a flow independent of required pressure. Examples are electric current generator and imposed velocity in mechanics. They are designated as given by the following figure.


FIGURE A.3. Representation of effort and flow sources in bond graph
Here the orientation of the half arrow is normally out of the sources.

## A.2. 5 Junctions

Some times called three ports, the junctions allow to couple the elements of a system, particularly $R, C$, and $I$-elements. They conserve power.

- Parallel junctions (0-junctions). They associate elements under the same effort, which means parallel circuit in electricity and oil hydraulics, and series circuit in mechanics. This corresponds to the Kirchhoff voltage law in electricity. The constitutive equation and representation of such junction are described by figure A.4.


$$
\begin{aligned}
& e_{1}=e_{2}=\ldots e_{n} \\
& \sum_{i=1}^{n} a_{i} f_{i}=0
\end{aligned}
$$

FIGURE A.4. Parallel junction or 0 -junction with $a_{i}=1$ for entering power and $a_{i}=-1$ for leaving power

- Series junctions (1-junctions): They associate elements under the same flow, which means series circuit in electricity and oil hydraulics. In mechanics, this means circuit in parallel, and corresponds to the Kirchhoff current law in electricity. They are described by figure A.5.


$$
\begin{aligned}
& f_{1}=f_{2}=\ldots f_{n} \\
& \sum_{i=1}^{n} a_{i} e_{i}=0
\end{aligned}
$$

FIGURE A.5. Serial junction or 1-junction with $a_{i}=1$ for entering power and $a_{i}=-1$ for leaving power

In both junction we have power conservation, which can be expressed as:

$$
\begin{equation*}
\sum_{i=1}^{n} e_{i} f_{i}=0 \tag{A.6}
\end{equation*}
$$

Formerly, the junctions have been called $p$ - and $s$-junctions by author Jean Thoma. This is better for generating computer code but has been abandoned for international standardization.

## A. 3 Bond Graph Elements with two Ports

These elements have two bonds or ports and comprise the transformer and the gyrator, a kind of over crossed transformer.

## A.3.1 Transformer TF

This element with two ports makes a model of an electric transformer, a gear reducer in rotary mechanics or simply a lever in rectilinear mechanics. The modulus of the transformer may not be constant, but depend on the time (or any other parameter). One calls it then $M T F$ (modulated transformer) and the transforming variable can be continuous or Boolean.


$$
\begin{aligned}
& e_{1}=m e_{2} \\
& f_{2}=m f_{1}
\end{aligned}
$$

FIGURE A.6. Transformer $T F$ and its constitutive equation

## A.3.2 Gyrator GY

Called also transducer $T D$, this two ports element is shown with the constitutive equations by the figure A.7.


FIGURE A.7. Gyrator and its constitutive equation
Examples are the gyroscope, electric motor. There exists also the modulated gyrator $M G Y$, which is the electric motor with variable excitation.

Let us note that transformer and gyrator are power conserving but can transform this power from one domain to the other. An example is the electric motor, that transforms from electric into mechanical rotary power.

## A.3.3 Carrousel of Thoma

The constitutive relations for different elements are well illustrated by the Thoma carrousel [Thoma, 1990] as presented by figure A.8. So a resistor or an $-R$-element gives an algebraic relation between effort and flow, an $I$-element between flow and integral of effort (or momentum $p$ ), and $C$-element between effort and integral of flow (or displacement $q$ ). Going around clockwise, we obtain only derivations, anti-clockwise integration's. Since all computers prefer integrations, we should always go anti-clockwise. Note that in the literature exist also the tetragon of Paynter, which is the same, but the name is not right, because it means the simplest body in three dimensions, limited by four triangles.


FIGURE A.8. Carrousel of Thoma

## A.3.4 Information Bonds

When one variable in a bond is very small it can be neglected, and the transmitted power is also negligible. We have then an information bond, which is the same as in all block diagrams. It is shown as a full arrow on
the bond and can represent the transmitted signal by a sensor, integrator, sum member, etc.. Figure A. 9 represents also the effort sensor $D e$ and the flow sensor $D f$.

(a)

(b)

FIGURE A.9. Information bond. a. transmitted signal, b. detector of effort and flow

## A. 4 Multiports Elements or C, I and R Fields

The bond graph elements so far have scalar constitutive laws. There exist also multiport elements, where each bond has several strands just like a vector (We avoid the name component, because this has an other meaning, the part of a machine). They have been treated in Karnopp 1975 and in the older book of Thoma 1975 and appear on figure A.10.

## Multiport C

This element shown by the figure A. 11 and called also formerly $C$-field is associated as energy storing by displacements and associated efforts.

The stored energy is

$$
\begin{align*}
E & =\int_{t_{0}}^{t} \sum_{i=1}^{n}\left(e_{i} f_{i}\right) d t=\int_{t_{0}}^{t} \sum_{i=1}^{n}\left(e_{i} \dot{q}_{i}\right) d t  \tag{A.7}\\
& =\int_{q_{0}}^{q} \sum_{i=1}^{n} e_{i}(q) d q_{i}=\int_{q_{0}}^{q} \sum_{i=1}^{n} e(q) d q=E(q)
\end{align*}
$$

where :

$$
\begin{aligned}
& q=\left[\begin{array}{lll}
q_{1} & \ldots & q_{n}
\end{array}\right]^{T} \\
& e
\end{aligned}=\left[\begin{array}{lll}
e_{1} & \ldots & e_{n}
\end{array}\right]^{T} .
$$

Note that here the efforts $e_{i}$ depend on all displacements. The relation can be linear, given by a matrix, or non linear, given by set of functions.

As an example, the energy stored in a thermodynamic and chemical system having $n$ chemical components is shown as figure A. 12 .

This multiport $C$ corresponds to the equation of Gibbs. This equation gives the stored energy $U$ as follows:

| System | Representation | constitutive equation |
| :---: | :---: | :---: |
| $\begin{aligned} & \left.\frac{e_{1}}{f_{l}} \begin{array}{l} e_{n} \\ \hline f_{n} \end{array}\right] \end{aligned}$ | $\frac{e}{f}$ |  |
| $\begin{aligned} & \hline e_{1} \\ & \hline f_{l} \\ & \frac{e_{n}}{f_{n}} 乙 \mathbf{J} \\ & \frac{e_{n+}}{f_{n=1}} \\ & \frac{e_{n+m}}{f_{n+m}} \end{aligned}$ | $\frac{e}{f(n)} \mathbf{J} \stackrel{e}{f(m)}$ |  |
|  | $\frac{e}{f(n)}-\stackrel{e}{f(m)}$ | $e_{i}^{T} f_{i}-e_{k}{ }^{T} \cdot k=0$ |
|  | $\frac{e_{l}(n)}{\Theta_{1}(n)}: \mathbf{T F} \frac{e_{2}(n)}{f_{2}(n)}$ | $\left\{\begin{array}{l}e_{l}^{T} f_{l}=e_{2}{ }^{T} f_{2} \\ e_{l}=m e_{2} \\ f_{l}=m f_{2}\end{array}\right.$ |
| $\begin{array}{lll} \hline e_{11} & \mathbf{G Y} & e_{21} \\ \hline f_{11} \\ e_{1 n} & \mathbf{r}_{1} & \overline{f_{21}} \\ \frac{\mathbf{G Y}}{} & e_{2 n} \end{array}$ | $\xrightarrow[f_{1}(n)]{e_{l}(n)}: \mathbf{R} \xlongequal[f_{2}(n)]{\mathbf{G Y}}$ | $\left\{\begin{array}{l} e_{1}^{T} f_{1}=e_{2}^{T} f_{2} \\ e_{1}=R f_{2} \\ e_{2}=R f_{1} \end{array}\right.$ |

FIGURE A.10. Multiport elements


FIGURE A.11. Multiport $C$


FIGURE A.12. Thermodynamic and chemical example of a multiport $C$

$$
d U=T d S-P d V+\sum_{i=1}^{n} \mu_{i} d n_{i}
$$

Here the displacement variables are volume $V$, entropy $S$ and the molar masses $n_{i}$. The corresponding efforts are pressure $P$, temperature $T$ and chemical potential $\mu$. Note the sign minus in front of the pressure translates as the hydraulic power orientation.

## Multiport I

The multiport $I$ corresponds to several inductors or masses in mechanics in interaction as shown on figure A. 13 .


FIGURE A.13. Multiport I
The multiport $I$ is the analogue to the multiport $C$, only effort and displacement are turned into momentum $p_{i}$ and flow $f_{i}$. Hence we have the equation:

$$
\begin{align*}
E & =\int_{t_{0}}^{t} \sum_{i=1}^{n}\left(e_{i} f_{i}\right) d t=\int_{0_{0}}^{t} \sum_{i=1}^{n}\left(f_{i} \dot{p}_{i}\right) d t=\int_{p_{0}}^{p} \sum_{i=1}^{n} f_{i}(p) d p_{i}  \tag{A.8}\\
& =\int_{p_{0}}^{p} \sum_{i=1}^{n} f(p) d p=E(p)
\end{align*}
$$

where

$$
\begin{aligned}
& p=\left[\begin{array}{lll}
p_{1} & \ldots & p_{n}
\end{array}\right]^{T} \\
& f=\left[\begin{array}{lll}
f_{1} & \ldots & f_{n}
\end{array}\right]^{T}
\end{aligned}
$$

Generally we have a matrix linking all displacements and efforts. The off-diagonal entries of the matrix indicate the coupling of variables.

## A.4.1 Mixed Multiport IC

We have also a combination between $C$ and $I$ multiport, having $n$ ports as multiport $C$ and $m$ ports as multiport $I$, as shown on figure A.14.


FIGURE A.14. Multiport $I C$
The stored energy is:

$$
\begin{gather*}
E=\int_{t_{0}}^{t} \sum_{i=1}^{n}\left(e_{i} f_{i}\right) d t=\int_{p_{0}}^{p} \sum_{i=1}^{k}\left(f_{i}\right) d p_{i}= \\
\int_{p_{0}}^{p} \sum_{i=1}^{n} f_{i}(p) d p_{i}+\int_{q_{0}}^{q} \sum_{j=k+1}^{n} e_{j} d q_{i} \tag{A.9}
\end{gather*}
$$

Here $p$ is the momentum vector and $q$ the displacement vector. The first $k$ ports represent a multiport $C$ and the multiports $I$, and the port from $k+1$ to $n$ represent the multiports $C$.

The multiport $I C$ is used mostly for electromagnetic effects with mechanical parts. Here the electric part is a multiport $I$ and the mechanical part a multiport $C$.

## A.4.2 Multiport $R$

This is essentially coupled resistor network, which links, in a algebraic relation between the $n$ flows and $n$ efforts without storing energy. The constitutive law can be linear, like Ohm's law or non linear, like the hydraulic resistors. In resistance causality (Fig. A.15a), we have:

$$
\begin{equation*}
e_{i}=\Phi_{R i}\left(f_{1}, \ldots f_{n}\right), i=1,2, \ldots n \tag{A.10}
\end{equation*}
$$

and in conductance causality (Fig. A.15ba),

$$
\begin{equation*}
f_{i}=\Phi_{R i}^{-1}\left(e_{1}, \ldots e_{n}\right) \tag{A.11}
\end{equation*}
$$

The multiport $R$ is generally symmetric in conductance and resistance causality. In thermodynamics, the corollary is the symmetry or reciprocity of Onsager.


FIGURE A.15. Multiport $R$

## A.4.3 Multiport RS

We know that the $R$-elements dissipate power and that this power comes out as heat. So including thermal effects, an $R$-element becomes an irreversible and power conserving structure. It is denoted as multiport $R S$ (Fig. A.16a). So power can flow only as indicated by the half arrows, and not backwards. In other words it cannot become negative. So, when we are not interested in thermal effects, we speak of $R$-elements and multiport- $R$, otherwise of multiport- $R S$.

Regarding the multiport $R S$, it can have bonds with several strands as shown on figure A.16b.

With multiport $R S$, irreversibility and energy conservation of multiport $R$ are as follows: with several strands, only the sum of the non thermal bonds must be positive, but in single strands power can become negative as long as it is more positive in others. One can also say that power in the thermal bond must by always positive.

(a)

(b)

FIGURE A.16. Element (a) and Multiport (b) $R S$

## A. 5 Causality Concept

One important structural propriety of the bond graph is its causality concept. Indeed, the determination of causes and effects in the system is directly deduced from the graphical representation and shows at the time the transition to the block diagram. The idea of bond graphs is to write it at first without causality, perhaps without power directions, and to worry about these later.

The relation between cause and effect on a bond graph are indicated by a vertical stroke. By convention, the side of the stroke indicates where the effort is acting, and the flow acts then in the reverse. In this sense, a causality stroke replaces two connections in opposed sense in a block diagram.

In our example given by the figure A.17a, the element $\mathbf{A}$ applies an effort on $\mathbf{B}$, which replies by a flow acting backwards from $\mathbf{B}$ to $\mathbf{A}$; so the effort from $\mathbf{A}$ is given from the outside to $\mathbf{B}$. In the corresponding block diagram the direction of action is indicated by an arrow on each connection as illustrated by figure A.17b. Independently of the causality, the direction of the positive power is indicated by the half-arrow on the bond.


FIGURE A.17. Causality
The notion of causality in bond graphs allows to resolve the algorithmic level of modelling. One tries to choose the causalities always integral and not derivative. Indeed, in numerical form, integration is simpler and more accurate than derivation. In other words, with the $C$-element the effort-out causality is preferred and with $I$ element the effort in causality is preferred.

As shown on the figure A.18, the bond graph symbol gives us four informations:

1. the existence of physical link between two systems by the bond,
2. the type of power (electric, mechanical...) by the power variables,
3. the power direction by the half arrow,
4. the causality by the stroke.

## A.5.1 Rules of Causality Selection

The choice of causality is free in principle but subject to some limitation : for example sources always impose an effort or a flow to the system.

Briefly they are:

- The sources impose always one causality, imposed effort by effort sources and imposed flow by flow sources.


FIGURE A.18. Informations given by bond graph representation

- With the one ports, a $C$-element has one (or several) flows as an input and gives effort as an output.
- The $I$-element takes one or several efforts as input and has flow as output. This choice results in integration of inputs to obtain the output. The reverse choice would give the undesirable derivation.
- In linear $R$-elements, the causality is in principle indifferent, but indicates whether resistance or conductance need to be entered as parameter. In non linear $R$-elements, equations are more comfortable in one direction, like our mass (or volume) flow and pressure characteristic in equation 2.11. There we have effort as input and flow as output.
- With the two ports, on the transformers $T F$, effort and flows point through and with gyrators $G Y$, an effort produces a flow; this in both directions.
- With the three ports (junctions), we have on the parallel junction (0junction) one input of effort, which is common to all bonds. It must be given by one bond, and all the other bonds have flow pointing to the junction and efforts going away.
- The series junction (1-junction), one flow, the common flow points to the junction, and all flows point away. In other word, all but one effort pushes to the series junction, and all other efforts push away.

From these considerations, the practical rules of causality application are [Karnopp 1975] [Thoma 1991] and [Borne et al. 1992]:

1. Apply a fixed causality to the source elements $S e$ and $S f$.
2. Apply a preferred causality to $C$ and $I$ elements.
3. Extend the causality through the nearly junction $, 0,1, T F$ an $G Y$.
4. Assign a causality to $R$ element which have indifferent causality .
5. If these operations give a derivative causality on one element, we have a causal conflict. One can handle derivative causalities (leading to so called $D A E=$ Differential Algebraic Equations). In our experience it is usually better to add small $C$ - or $I$-elements to avoid causal conflicts; often enough physical reasons can be found for such elements like small dead volumes. Naturally, such small $C$ - or $I$-elements increase the state space of the system, but with modern computers, that is less a problem. Often they do not modify measurably the dynamics of the system. An can also make the small $C$-elements larger, say ten times as large do so called "remedy of the bad conscience". Bad conscience because we do not know for sure what the influence on the visible system dynamics will be.

The causalities of bond graph elements are given by the figure A. 19 and A. 20 .

## A. 6 Bicausal Bond Graphs

As we have seen, causality imposes a certain propagation throughout the bond graph. It implies that if effort acts in one sense, flow acts in the reverse which is true if we know then values of the bond graph elements. If not, other rules are necessary. One generalization of the causality selection proposed by [Gawthrop, 1985], the bicausal bond graph. There one divides the causality stroke into two. One half indicates the direction of effort, the other the direction of flow as shown on figure A.21.

On the figure A.21, the causality of $R$ indicates that both effort and flows enter the $R$-element. This allows to calculate the resistance $R$. Causality on the effort source indicates, that the effort is imposed, and the flow is known, for example by a flow sensor. If one affects bicausality to a $C$-element, an effort sensor must be added on the element.

## A. 7 State Space Equations Associated to a Bond Graph

The state vector, denoted by x , is composed by the variables p and q , the energy variables of $C$ - and $I$-elements.

$$
x=\left[\begin{array}{l}
p_{I} \\
q_{C}
\end{array}\right]
$$

| Element | Causal form | Relation | Block diagram | Rule |
| :---: | :---: | :---: | :---: | :---: |
| Effort <br> source (Se) <br> and <br> flow <br> source (Sf) | $\begin{aligned} & \text { Sf } \frac{\mathrm{e}}{\mathrm{f}} \\ & \operatorname{Se}^{\mathrm{e}} \frac{\mathrm{e}}{\mathrm{f}} \end{aligned}$ | fgiven <br> e given | $\begin{aligned} & \text { Sf } \xrightarrow{\stackrel{e(t)}{f(t)} \text { systeme }} \\ & \text { Se } \xrightarrow{\frac{f^{(t)}}{e(t)}} \text { systeme } \end{aligned}$ | Effort or flow source given. <br> The causality is obligatory (required). |
| 0 Junction |  | $\begin{aligned} & e_{2}=e_{1} \\ & e_{3}=e_{1} \\ & e_{4}=e_{1} \\ & f_{1}=-\left(f_{2}+f_{3}+f_{4}\right) \end{aligned}$ |  | Only one effort gives its value to the others, (here $e_{1}$ ). Only one causal stroke on 0 junction. |
| 1 Junction | $\stackrel{e_{2} \mid f_{2}}{\mathrm{e}_{1}} \stackrel{\frac{\mathrm{f}_{1}}{1}}{\substack{\mathrm{f}_{1}}} \stackrel{\mathrm{e}_{3}}{\mathrm{f}_{4} \mid e_{4}}$ | $\begin{aligned} & f_{2}=f_{1} \\ & f_{3}=f_{1} \\ & f_{4}=f_{1} \\ & e_{1}=-\left(e_{2}+e_{3}+e_{4}\right) \end{aligned}$ |  | Only one flow gives its value to the others, (here $f_{1}$ ). <br> Only one bon with out causal stroke |
| transformer $T F$ | $\left\lvert\, \begin{array}{ll} \mathrm{e}_{1} & \text { TF } \\ \mathrm{f}_{1} & : \mathrm{m} \mathrm{l}_{\mathrm{f}_{2}} \end{array}\right.$ | $\begin{aligned} & e_{1}=m e_{2} \\ & f_{2}=m f_{1} \end{aligned}$ |  | $e_{2}$ and $f_{1}$ ar known. <br> Only one causal stroke on TF |
| $T F$ |  | $\begin{aligned} & f_{1}=f_{2} / m \\ & e_{2}=e_{l} / m \end{aligned}$ |  | $e_{1}$ and $f_{2}$ ar known. <br> Only one causal stroke on TF |

FIGURE A.19. Causalities of bond graph elements (part 1)
Properties of State Variables

- The state vector does not appear on the bond graph, but only its derivative

$$
\dot{x}=\left[\begin{array}{c}
e_{I} \\
f_{C}
\end{array}\right]=\left[\begin{array}{c}
\dot{p}_{I} \\
\dot{q}_{C}
\end{array}\right]
$$

- The dimension of the state vector is equal to the number of $C$ - and $I$-elements in integral causality
- If among the $n C$ and $I$-elements, $n_{l}$ are in derivative causality, so the order of the model is $n-n_{l}$. The sate vector is then split up into the parts $x_{i}$ and $x_{d}$ ( $i$ for integral and $d$ for derivative).

| Element | Causal form | Relation | Block diagram | Rule |
| :---: | :---: | :---: | :---: | :---: |
| Gyrator |  | $\begin{aligned} & e_{1}=r f_{2} \\ & e_{2}=r f_{1} \end{aligned}$ | $\begin{array}{rl} e_{1} \longrightarrow & 1 / \mathrm{r} \\ \mathcal{F}_{1} & 1 / \mathrm{r} \\ f_{2} \end{array}$ | $f_{2}$ and $f_{1}$ are given for GY No causal stroke on GY |
| GY | $\left\|\frac{e_{1}}{f_{1}}{ }^{\text {G/r }} \underset{: r}{G Y}\right\|_{f_{2}}^{e_{2}}$ | $\begin{aligned} & f_{1}=e_{2} / r \\ & f_{2}=e_{1} / r \end{aligned}$ |  | $e_{2}$ and $e_{1}$ are given for GY 2 causal strokes on GY |
| Resistance R |  | $e=\phi_{r}()$ | ${ }^{e} \stackrel{D_{\mathrm{R}}}{ }$ | $f$ is given for R Indiff. causality |
|  | $\frac{e}{f}{ }^{\text {c }} \mathbf{} \mathbf{R}$ | $f=\phi_{R}^{-1}(e)$ | $\xrightarrow[f]{e} \longrightarrow e=\frac{d}{d t} \Phi_{+}(f)$ | $e$ is given for R . Indifferent causality |
| Capacity | $\frac{e}{f=\dot{q}} \text { C }$ | $e=\phi_{C}^{-1}\left(\int f d t\right)$ |  | $e$ is given for C <br> Derivative <br> causality |
| C | $\frac{e=\dot{p}}{f}>\mathrm{C}$ | $f=\frac{d}{d t} \phi_{C}(e)$ |  | $e$ is given for C-element. Derivative causality |
| Inertia | $\frac{e=\dot{p}}{f} \mathcal{I I}_{\text {I }}$ | $f=\phi_{l}^{-1}\left(\int e d t\right)$ | $\begin{aligned} & e \\ & f \\ & \hline \end{aligned}$ | $e$ is given for I-element. Integral causality |
|  | $\stackrel{e=\dot{p}}{f} \mathbf{I}$ | $e=\frac{d}{d t} \phi_{l}(f)$ |  | $f$ is given for I-element. Derivative causality |

FIGURE A.20. Causalities of bond graph elements (part 2)

$e_{1=} e_{2}, \quad f_{1=} f_{2}$

$e_{2}=e_{1}, \quad f_{2}=f_{1}$

FIGURE A.21. Bicausal bond graph

## A. 8 Pseudo Bond Graphs

The true bond graphs, as shown above on the table 4.1 have the physical power as product of effort and flow. Precisely, it is the instant power travelling along a bond or entering a port. Such bond graphs are well suited for mechanical., hydraulic and electric systems.

In the pseudo bond graphs [Karnopp, 1979], the product effort-flow has no more the meaning of a power, it has no meaning in general. Such bond graphs are more necessary for fluid lines, as treated in section 2.1, but also in physical chemistry, diffusion and chemical reactions. We have then pseudo bonds, but the rules of causalities and signs remain valid in $C, I$ and $R$-elements.

One finds the pseudo bond graphs in the following fields:

1. In thermal conduction and engineering, one uses the heat flow and not of entropy flow. This is because heat flow is conserved in such problem and allows to write a simple $R C$ circuit for such problems. Entropy flow is not conserved and increases generally with decreasing temperature.
2. In chemistry, one uses often not a chemical potential (which forms with molar flow a true bond graph) but the concentration. Their relation is explained in section 3.1 and formula 3.49.
3. In compressible fluid mechanics, there are conduction phenomena. So one use as flows the mass flow and the enthalpy flow because they have simple conservation principles. Mass flow is always conserved, except in leakage, which is reticulated be a parallel junction and an $R$-element. Enthalpy flow can be derived from a power balance." The true Bond graphs are a virtue, but the pseudo Bond graphs are a necessity".

## A. 9 Conservation of Power and Conservation of Energy

For the junctions, the transformer and the gyrator, there is conservation of power and therefore conservation of energy. There is in bond graph another conservation of energy in the elements $C$ and $I$. These elements do not conserve power but can absorb it. Then they change their state and in order to return to the initial energy one must take out exactly the same energy. Hence, energy is conserved over one cycle. Therefore, it is of advantage to make a difference between power conservation and energy conservation.

In the multiports $C$ and $I$, energy conservation is different. It can go out though another bond, but the total energy is always the same to return to the initial state.

The $R$-element is irreversible and absorbs but give out power. In the multiport $R$, the condition is weaker: The sum of powers is always positive, but some power may come out from one bond, be negative, as long as the sum over all bonds remains positive. One application is the thermoelectricity and the Peltier effect [Thoma, 1975].

Any multiport that has a more complex structure will be represented in this book by four or five letters, the symbols with one to three letters reserved for standard bond graph elements. Examples are the designations Reco, Hexa and Tefma, which designate resistance for convection, heat exchanger and thermal turbomachine; they are developed in chapter 2.

## A. 10 Thermal Inertia and Second Law of Thermodynamics

The thermodynamic literature [Cellier, 1994] says that a thermal element $I$ can not exist because it would be contrary to the second law of thermodynamics. This not the case as we shall show below.

Figure A. 22 shows two pistons in two cylinders, which are in communication, not by a leakage with mass flow, but simply by a thermal conduction with temperature and entropy flow. The pistons serve only to vary pressure and volume and therefore the temperature according to the constitutive equation of the enclosed gas. Each change of the temperature generates a flow of entropy from the higher to the lower temperature and becomes zero when equality of temperatures is obtained. A new position of the pistons introduces a new difference of temperature somewhat similar to the electric circuit of section 3.3.

The figure A.22b shows the corresponding bond graph. Let us note, that here we have a true bond graph with $T$ and $\dot{S}$ and not a pseudo-bond graph, which is better for simulation. Since we are already in the thermal domain, the new entropy of the multiport $R S$ is injected by the switch $S W I T$ in the bond of lower temperature. The second law of thermodynamics says that this dissipation is always positive and that the new entropy can not be destroyed.

One arguments now, that an element $I$ in series on figure A. 23 can displace entropy from lower to higher temperature which is not according to the second law. Consequently a thermal $I$-element can not exist.
It is true that $I$-element can pump the entropy from the lower to higher temperature but this supposes that the stored energy in this element is consumed and it can work only as long as there is some energy. Therefore the energy for charging the element $C$-comes from the element $I$ and that


FIGURE A.22. Thermal conduction between two cylinders. a. Schema, b. bond graph model


FIGURE A.23. Bond graph model of thermal conduction between two Cylinders with a thermal I-element
is perfectly possible. As always the elements $I$ and $C$ are reversible and the only irreversibility is in the multiport $R S$.

The error comes from the fact, that the second law as the first law is applicable only to the continuous movement or steady state. In discontinuous or unsteady process, the first law is also not obeyed as show all play toys with a spring that produces mechanical energy.

## A. 11 Inverse Analogy

It should be mentioned for completeness, that there exist also mechanical bond graphs, where the role of effort and flow is interchanged. In other words, velocity is considered an effort and force a flow, or similarly rotation frequency an effort and torque a flow. We call this the inverse analogy. It is not used by practical bond graphers but has some theoretical advantages. It relates to an other graph theoretical method, the linear graphs.

With linear graphs, one divides the variables not in effort or flow, but in across and through variables. An across variable can be measured by applying a voltmeter, in the electrical case, on two bonds adjacent to an element, while a trough variable requires cutting a link, to insert an Ammeter. Writing now a bond graph in the inverse analogy has the effect that all efforts are across variables and all flows are through variables. Then graph theoretical methods can be used and such a bond graph is easily converted to a linear graph.

Author Jean Thoma has discussed the question of analogies with Henry Paynter at several times. Our conclusion was there is no theoretical reason to prefer one analogy or the other, both work and give computable bond graphs. However, the inverse analogy goes against the picture of electricity of Faraday and Maxwell, where one speaks of electromagnetic force and not about electromagnetic flow. Therefore all practical bond graphers including the authors use the normal analogy as expounded here.

## A. 12 References

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[Ganwtrop, 1995] Gawthrop P.J. "Bicausal Bond Graphs" International confernce on Bond Graph Modeling and Simulation" (IBGM'95), pp. 83-88, Las Vegas, USA, 1995.
[Karnopp, 1975] Karnopp D. C. and R.C. Rosenberg "Systems dynamics. A unified Approach", Wileey Intersciences; New York, 1975.
[Karnopp, 1979] Karnopp, D. K, "State Variables and PseudoBond Graphs for Compressible Thermofluid Systems", Trans. ASME J. DSMC, Vol. 101, pp. 201-204, 1979.
[Thoma, 1975] Thoma, J. U., "Introduction to Bond graphs and their Applications", Pergamon Press, 1975.
[Thoma, 1991] Thoma, J. U., "Simulation by Bond graphs", Springer Verlag 1991.

## Appendix B

## Listing of Simulation Programs

B. 1 Hydro-Pneumatic Accumulator (Tutsim)<br>$=$ Timing: 0.0020000 ,DELTA ; 10.0000 ,RANGE<br>$=$ PlotBlocks and Scales:<br>=Format:<br>= BlockNo, Plot-MINimum, Plot-MAXimum; Comment<br>=Horz: $0,0.0000,10.0000$; Time<br>$=\mathrm{Y} 1: 23,0.0000,10.0000$; gas pressure<br>$=\mathrm{Y} 2: 31,273.0000,373.0000$; tempera gas<br>$=$ Y3: $33,273.0000,373.0000$; heat capac wall<br>$=$ Y4: $35,0.0000,500.0000$; ext heat conduc<br>12 SF $121 \mathrm{P} 1=2.0000$;Frequency excitation<br>$14 \mathrm{I}-1519 \mathrm{P} 1=300.0000 ;$ mass of $1 / 4$ car $\mathrm{P} 2=0.0000$<br>15 CON P1 $=3.000 \mathrm{E}+03$;gravity<br>$19 \mathrm{TF} 2223 \mathrm{P} 1=1.000 \mathrm{E}+03$; piston press area<br>$21 \mathrm{TF} 12-14 \mathrm{P} 1=1.000 \mathrm{E}+03$;pisto area (flow)<br>22 R $21 \mathrm{P} 1=0.0500000$;damping resist<br>23 DIV 413212 ;gas pressure<br>31 ATT $412 \mathrm{P} 1=1.4000$;tempera gas<br>32 R $31-33 \mathrm{P} 1=10.0000$;int heat conduct<br>33 C $32-35 \mathrm{P} 1=10.0000$;heat capac wall $\mathrm{P} 2=298.0000$<br>35 R $33-36 \mathrm{P} 1=2.0000$;ext heat conduc<br>36 CON P1 $=298.0000$;ambient temper<br>41 MUL 2123 ;compression power<br>121 FRQ P1 $=0.5000000$;frequency genera $\mathrm{P} 2=0.0000$<br>211 INT $21 \mathrm{P} 1=0.0000$;calcul volumen<br>212 SUM - 211213 ;effect volu<br>213 CON P1= 120.0000 ;init volu<br>221 MUL 2122 ;damping power<br>411 INT 22141 -32 P1= 300.0000 ;calcul intern enrgy<br>412 ATT $411 \mathrm{P} 1=0.7180000$;divi by Cv<br>413 GAI $411 \mathrm{P} 1=0.4000000$; mult by $\mathrm{R} / \mathrm{Cv}$

## B. 2 Refueling Pneumatic Automobiles (Tutsim)

```
=Timing: 0.0040000 ,DELTA ; 100.0000 ,RANGE
    =PlotBlocks and Scales:
```

=Format:
= BlockNo, Plot-MINimum, Plot-MAXimum; Comment
=Horz: $0,0.0000,100.0000$; Time
$=\mathrm{Y} 1: 13,0.0000,20.0000$; pressu right
$=\mathrm{Y} 2: 23,0.0000,20.0000$; pressu left
= Y3: $14,200.0000,400.0000$; tempera bootle 1
= Y4: $24,200.0000,400.0000$; tempera bottle 2
1 CON P1 $=0.0000$;air bottle filling
2 CON P1 $=0.0000$;fully chokd flow by FNC234
3 CON P1 $=0.0000$;scanvening by mdot41
13 ATT $323 \mathrm{P} 1=3.000 \mathrm{E}+03$;pressu right
14 DIV 322311 ;tempera bootle 1
23 ATT 623 P1 = 300.0000 ;pressu left
24 DIV 622611 ;tempera bottle 2
31 MUL 13113291234 ; mass flow
32 GAI 325 P1= 1.0050 ;enthalpy flux
41 GAI $46 \mathrm{P} 1=1.0000$;mass flow pump
42 GAI 241 P1 $=1.0050$;enthalpy flow pump
46 FNC 999
X,Y-1 0.00000 .0000
X,Y- 215.00000 .0000
X,Y-3 15.10001 .0000
X,Y- 490.00001 .0000
X,Y-5 90.10000 .0000
X,Y- 6200.00000 .0000
91 CON P1= 113.0000 ;window area
131 DIV 13141 ;divi by root tempera
132 CON P1 $=0.0600000$;flow constant
141 SQT 14 ;root temperature
234 FNC 235
X,Y- 10.00000 .5000000
X,Y- 20.50000000 .5000000
X,Y- 30.60000000 .4900000
X,Y- 40.70000000 .4580000
X,Y-5 0.80000000 .4000000
X,Y- 60.90000000 .3000000
X,Y-7 0.95000000 .2180000
X,Y- 81.00000 .0000
235 DIV 2313 ;pressu ratio
241 MUL 2441 ;part enthalpy flow pump
311 INT - $3141 \mathrm{P} 1=770.0000$;mass left bottle
321 INT - $3242 \mathrm{P} 1=150.000 \mathrm{E}+03$;inter ebrgy left
322 ATT $321 \mathrm{P} 1=0.7180000$; left int energy divi Cv
323 GAI $321 \mathrm{P} 1=0.4000000$; divi $\mathrm{R} / \mathrm{Cv}$ right
325 MUL 3114 ;mult by temperature

611 INT $31-41 \mathrm{P} 1=19.3000$;mass right bottle
621 INT $32-42 \mathrm{P} 1=3.750 \mathrm{E}+03$;inter enrgy right
622 ATT $621 \mathrm{P} 1=0.7180000$;right int energy divi Cv
623 GAI $621 \mathrm{P} 1=0.4000000$; divi $\mathrm{R} / \mathrm{Cv}$ left

## B. 3 Cooling of fuel for Diesel Cars (Tutsim)

$=$ Timing: 0.0020000 ,DELTA ; 5.0000 ,RANGE
$=$ PlotBlocks and Scales:
= BlockNo, Plot-MINimum, Plot-MAXimum; Comment
=Horz: $0,0.0000,5.0000$; Time
= Y1: $32,0.0000,0.0500000$; mass flow of hexa 2
$=\mathrm{Y} 2: 13,0.0000,1.0000$; pressu before Hexa 3
$=\mathrm{Y} 3: 14,0.0000,1.0000$; pressu before canister 4
$=\mathrm{Y} 4: 11,0.0000,1.0000$; pressu before Hexa 1
,Fuel circuit with IP (Injection Pump), 3 hexa, canister and tank ,Multi-BG
with thermal bonds with md and Hd
11 ATT 311 P1= 5.0000 ;pressu. before Hexa 1
12 ATT 321 P1= 1.0000 ;pressu. before Hexa 2
13 ATT $331 \mathrm{P} 1=1.0000$;pressu. before Hexa 3
14 ATT $341 \mathrm{P} 1=5.0000$;pressu before canister 4
15 ATT $351 \mathrm{P} 1=1.0000$;pressu. in tank 5
21 ATT 412 P1 $=1.0000$;temp. in IP 1 input
22 ATT $422 \mathrm{P} 1=1.0000$;temp in Hexa 2 input
23 ATT $432 \mathrm{P} 1=1.0000$;temp in Hexa 3 input
24 ATT 442 P1 $=1.0000 ;$ temp in Canister 4 input
25 ATT $452 \mathrm{P} 1=1.0000 ;$ temp in tank 5
28 CON P1 $=0.0000$;enviromental temperature
31 ATT $11-12 \mathrm{P} 1=5.0000$;mass flow in IP 1
32 ATT $12-13 \mathrm{P} 1=0.2000000$;mass flow of hexa 2
33 ATT $13-14 \mathrm{P} 1=5.0000$;mass flow of hexa 3
34 SF $93 \mathrm{P} 1=0.5000000$;mass flow tank to canister
36 ATT $14-11 \mathrm{P} 1=0.2000000$;mass flow hexa 6
41 CON P1 $=0.0000$;enthalpy inflow IP
42 GAI $221 \mathrm{P} 1=1.0000$;enthalpy in hexa 2 input
43 GAI 241 P1=1.0000;enthalpy in Hexa 3 input
44 GAI $251 \mathrm{P} 1=1.0000$;enthalpy flow tank to canister
46 GAI $241 \mathrm{P} 1=0.0000$;enthalpy hexa 6
51 CON P1 $=0.0000 ;$ add mech energy
82 ATT $22-28 \mathrm{P} 1=5.0000$;therm conduc hexa 2 w enviro
83 ATT $23-28 \mathrm{P} 1=5.0000$;therm conduc hexa 3 w enviro
84 ATT $24-25 \mathrm{P} 1=5.0000$;therm conduc tank to can
85 ATT $24-28$ P1 $=5.0000$;therm condu tank 5 w enviro

86 ATT 21 - 28 P1= 5.0000 ;therm condu hexa 6 w enviro
93 CON P1= 1.0000 ;control signal
211 MUL 3121 ;mass flow * temp21 of IP
212 GAI $211 \mathrm{P} 1=1.0000$;enthalpy without mech power
221 MUL 2232 ;prod mass flow*tempera hexa 2
241 MUL 2434 ;prod mass flow*tempera hexa 6
251 MUL 2534 ;prod mass flow*tempera canister
311 INT $36-31 \mathrm{P} 1=0.5000000$;mass in IP input
312 CON P1 $=0.5000000$;torque signal phi
313 CON P1= 1.0000 ;one for 1-phi
314 SUM -312 313 ;parameter 1-phi for spilled flow
319 MUL 31431 ;mass spill flow
321 INT $319-32 \mathrm{P} 1=1.0000$;mass in hexa 2
331 INT $32-33 \mathrm{P} 1=1.0000$;mass in hexa 3
341 INT 33-34-36 P1=1.0000;mass in hexa 4
351 INT - $34 \mathrm{P} 1=10.0000$;fuel mass in tank
361 INT $33-8444 \mathrm{P} 1=10.0000$;mass in hexa 6
411 INT $46941 \mathrm{P} 1=0.0000$;mass in IP 1
412 DIV 411311 ;enthalpy mass rat in IP 1
419 MUL 31441 ;spilled enthalgy flow
421 INT $419-42 \mathrm{P} 1=1.0000$;stored energy hexa 2
422 DIV 421321 ;enthal/mas ratio hexa 2
429 SUM $42-82$;enthalpy in output hexa 2
431 GAI 24133 P1= 1.0000 ;stored energy hexa 3
432 DIV 431331 ;enthal/mass ration hexa 3
439 SUM 43-83;enthalpy in output hexa 3
441 INT $439-44-36 \mathrm{P} 1=1.0000$;int,stored energy hexa 4
442 DIV 441341 ;enthalpy/mass ratio in canister 4
451 INT $4484-85 \mathrm{P} 1=10.0000$;stored enthalpy in tank
452 DIV 451351 ;enth/mass ratio in tank 5
469 SUM 46-86;enthalpy output hexa 6

## B. 4 Ventilation of a building (Twentesim)

## - 20-sim Experiment Description

Model: vent3a1
Experiment: $\exp 1$
Date: 08/31/99
Time: 15:34:57
Version: PC Version 2.3
License: Prof. G. Dauphin-Tanguy

+ cole Centrale de Lille, LAIL


## Parameters:

= = = = = = = = = =
data2d'filename 1.98855e-307
data2d'col 1
data2c'filename $1.98855 \mathrm{e}-307$
data2c'col 1
data2b'filename 1.98855e-307
data2b'col 1
data2a'filename 1.98855e-307
data2a'col 1
data1b'filename 1.98855e-307
data1b'col 1
data1a'filename 1.98855e-307
data1a'col 1
vent_3‘R13 0.01
vent_3‘Tref 273
vent_3‘Cp 0.001005
vent_2‘R13 0.01
vent_2'Tref 273
vent_2'Cp 0.001005
vent_1'R13 0.01
vent_1'Tref 273
vent_1'Cp 0.001005
fanx'R37 0.01
fanx'Tref 273
fanx'Cp 0.001005
fanx'filename $1.98855 \mathrm{e}-307$
fanx'col 1
r_362'R 25
r_361'R 25
r_262'R 25
r_261'R 25
r_162'R 25
r_161'R 25
r_35'R 50
r_25'R 50
r_15'R 50
c_36'c1 36
c_26'c1 36
c_16'c1 36
C2'Cv 0.000718
C2'Tref 0
C2'R 0.000287
C2'Pref 0
C2'V 60

C1‘Cv 0.000718
C1'Tref 0
C1'R 0.000287
C1'Pref 0
C1'V 60
Initial Conditions:
$===============$
c_36'state 0
c_26'state 0
c_16'state 0
C2'mdot_state 78500
C2'hdot_state 0
C1'mdot_state 78500
C1'hdot_state 0
Run Specifications:
==================
Integration Method Euler
Start Time 0
Finish Time 100
Step Size 0.01
Multiple Run Specifications:
===========================
Multiple Run Type : Plain Multiple Run
Number Of Steps : 2
Plot Specifications:
===================
X-Axis Time 0100
A C1'hdot_in.e-11
B C2'hdot_in.e-1 1
C C2'mdot_in.e -0.5 0.5
D fanx'hdot_out.f 050

## - Two ports C:C2

class 2 c version 2
interface
ports: hdot_in,mdot_in
orientation restrictions
fixed in hdot_in,mdot_in
causality restrictions
fixed effort hdot_in,mdot_in
parameters
real $\mathrm{Cv} \# 0.718 \mathrm{e}-3[\mathrm{KJ} / \mathrm{kgr}]$
real Tref \# Reference termperature, not mentioned in Tutsim Listing so take: $0[\mathrm{~K}]$
real R \# 0.287e-3
real Pref \# Reference Pressure, not mentioned in Tutsim Listing so take: 0 [kPa]
real V \# Room volume 60 [m3]
variables
real mdot_state \# initial value 78.5 e 3 [gr]
real hdot_state \# initial value 0
equations
mdot_state $=\operatorname{int}($ mdot_in.f) \# int141
hdot_state $=\operatorname{int}($ hdot_in.f) \# int131
mdot_in.e $=($ hdot_state*R $) /\left(\mathrm{Cv}^{*} \mathrm{~V}\right)$ - Pref
hdot_in.e $=($ hdot_state $/ \mathrm{Cv}) / \mathrm{mdot} \_$state - Tref

## - Two ports C:C1

class cdouble version 1
interface
ports: hdot_in,mdot_in
orientation restrictions
fixed in hdot_in,mdot_in
causality restrictions
fixed effort hdot_in,mdot_in
parameters
real $\mathrm{Cv} \# 0.718 \mathrm{e}-3[\mathrm{KJ} / \mathrm{kgr}]$
real Tref \# Reference termperature, not mentioned in 20sim Listing so take:
$0[\mathrm{~K}]$
real R \# 0.287e-3
real Pref \# Reference Pressure, not mentioned in 20sim Listing so take: 0 [ kPa ]
real V \# Room volume 60 [m3]
variables
real mdot_state \# initial value 78.5 e 3 [gr]
real hdot_state \# initial value 0
equations
mdot_state $=\operatorname{int}($ mdot_in.f) \# int141
hdot_state $=\operatorname{int}($ hdot_in.f) \# int131
mdot_in.e $=\left(\right.$ hdot_state*R) $/\left(\mathrm{Cv}^{*} \mathrm{~V}\right)$ - Pref
hdot_in.e $=($ hdot_state $/ \mathrm{Cv}) / \mathrm{mdot}$ _state - Tref

- Fanx1
class fanx version 1
interface
ports: mdot_in, hdot_in, hdot_out
causality restrictions
fixed flow mdot_in

```
fixed flow hdot_in
fixed flow hdot_out
orientation restrictions
fixed in mdot_in
fixed in hdot _in
fixed out hdot_out
parameters
real R37 # Hydraulic resistance for mdot, 0.01 [sec*kPa/gr]
real Tref # Reference temperature, 273 [K]
real Cp # Specific heat for hdot, 1.005e-3 [kJ/kgr]
string filename # parameter for filename table
real col # columnnumber of data for table
variables
real data3,var1,var2
equations
data3 = from_file(filename,col) # Equal to Se_370
mdot_in.f = (mdot_in.e + data3)/R37
var1 = if mdot_in.f <= 0 then hdot_in.e else hdot_out.e
var2 = Cp*mdot_in.f*(Tref + var1)
hdot_in.f = var2
hdot_out.f = var2
```

- Vent1

```
class vent version 1
interface
ports: mdot_in, hdot_in, hdot_out
causality restrictions
fixed flow mdot_in
fixed flow hdot_in
fixed flow hdot_out
orientation restrictions
fixed in mdot_in
fixed in hdot in
fixed out hdot_out
parameters
real R13 # Hydraulic resistance for mdot, 0.01 [sec*kPa/gr]
real Tref # Reference temperature, 273 [K]
real Cp # Specific heat for hdot, 1.005e-3 [kJ/kgr]
variables
real var1,var2
equations
mdot_in.f = mdot_in.e/R13 # Note that I use the reciprocal equation
var1 = if mdot_in.f <= 0 then hdot_in.e else hdot_out.e
var2 = Cp*mdot_in.f* (Tref + var1)
```

hdot_in.f = var2
hdot_out.f $=$ var2

## B. 5 Instrumentation of a Steam generator

| Sensor | Parameter | Unit | Value |
| :--- | :--- | :--- | :--- |
| L1 | Level in the tank | L | 200 |
| T2 | Temperature of water | ${ }^{\circ} \mathrm{C}$ | 37 |
| F3 | Flow of feedwater | $\mathrm{L} / \mathrm{h}$ | 950 |
| Q4 | Heat power | kW | 60 |
| T5 | Temperature of water in the boiler | ${ }^{\circ} \mathrm{C}$ | 168 |
| T6 | Temperature of vapour in the boiler | Bar | 170 |
| P7 | Pressure in the boiler | Bar | 8 |
| L8, L9 | Level of water | L | 150 |
| F10 | Flow of Outlet steam expansion | $\mathrm{kg} / \mathrm{h}$ | 83 |
| P11 | Pressure depending on F10 | Bar | 8 |
| P12, P13 | Upstream, downstream pressure in branche 1 | Bar | 8 |
| P14, P15 | Upstream, downstream pressure in branche 2 | Bar | 5 |
| P16 | Inlet pressure to condenser | Bar | 5 |
| T17 | Temperature at entrance of condenser | ${ }^{\circ} \mathrm{C}$ | 120 |
| VM1, VM2 | Control valve | $\%$ | - |
| F23 | Flow of Inlet cooling water | $\mathrm{kg} / \mathrm{h}$ | 2600 |
| L18, L19 | Level of condensate | $\%$ | 50 |
| T20 | Temperature output of condenser | ${ }^{\circ} \mathrm{C}$ | 52 |
| T21 | Temp. of inlet cooling water | ${ }^{\circ} \mathrm{C}$ | 40 |
| T22 | Temp. of outlet cooling water | ${ }^{\circ} \mathrm{C}$ | 70 |
| V3-V5 | Control valves |  |  |
| F24 | Flow cooling water in aerorefrigerator | $\mathrm{kg} / \mathrm{h}$ |  |
| T25 | Temp. of inlet cooling water | ${ }^{\circ} \mathrm{C}$ | 70 |
| T26 | Temp. of outlet cooling water | ${ }^{\circ} \mathrm{C}$ | 30 |
| T27 | Ambient temp. | ${ }^{\circ} \mathrm{C}$ | 20 |

TABLE B.1. Instrumentation of steam generator of figure 5.15

## Appendix C

## Polynomial thermodynamic Functions

1. Enthalpy of steam function as function of pressure

$$
\begin{equation*}
h_{v}(P)=-0,74 P^{2}-17,21 P_{3}+2680 \tag{C.1}
\end{equation*}
$$

2 Enthalpy of liquid as function of pressure

$$
\begin{equation*}
h_{l}(P)=-0,0243 P^{4}+0,8487 P^{3}-11,9 P^{2}-99.97 P+347 \tag{C.2}
\end{equation*}
$$

3 Specific volume of steam as function of pressure

$$
\begin{align*}
\nu_{v}(P)= & -5,3 * 10^{-5} P^{5}+0,00207 P^{4}-0.032 P^{3} \\
& +0,2498 P^{2}-1,03 P+2,166 \tag{C.3}
\end{align*}
$$

4 Specific volume of liquid as function of pressure

$$
\begin{equation*}
\nu_{l}(P)=-3,59 * 10^{-7} P^{3}+1,2456 * 10^{-5} P^{2}+1,039 * 10^{-3} \tag{C.4}
\end{equation*}
$$

5 Temperature as function of pressure

$$
\begin{equation*}
f_{T}(P)=T=0,459 . P^{2}+12,7243 . P+99,005 \tag{C.5}
\end{equation*}
$$

6 Partial derivative of mass density of steam at saturation as function of temperature

$$
\begin{align*}
d R h o=\left(\frac{\partial \rho_{3}}{\partial T}\right)_{T=T_{S A T}} & =5 * 9,18 * 10^{-12} T^{4} \\
& +1,59159 * 10^{-9} T^{3}  \tag{C.6}\\
& +3 * 2,6823812 * 10^{-7} T^{2} \\
& +2 * 5,672779398 * 10^{-6} T \\
& +1,9036631098 * 10^{-4}
\end{align*}
$$

7 Latent heat of steam as function of temperature

$$
\begin{align*}
& L_{V}(T)=-3,4 * 10^{-11} \cdot T^{4}+4,421 * 10^{-11} \cdot T^{3} \\
& \quad-1,54503768 * 10^{-6} \cdot T^{2}-2,21405632169 * 10^{-3} \cdot T  \tag{C.7}\\
& \quad+2,49715620119075 * 10^{6}
\end{align*}
$$

8 Enthalpy of liquid as function of temperature

$$
\begin{equation*}
h_{L}(T)=4,337 \cdot T-1,2025 * 10^{3} \tag{C.8}
\end{equation*}
$$

9 Enthalpy of steam as function of temperature

$$
\begin{equation*}
h_{V}(T)=1,1248 T+2267,5 \tag{C.9}
\end{equation*}
$$

Remark 1 Used units:
Enthalpy $h$ in $k J / k g$, Temperature in ${ }^{\circ} C$, Pressure $P$ in $b a r$, specific volume in $\mathrm{m}^{3} / \mathrm{kg}$.

## Appendix D

## Heat Losses and Heat Exchanger Coefficients of Collectors

We shall give here the main empirical relations for the calculation of thermal losses in steam generators. Since the theoretical determination is complex, one uses generally empirical relations for the parameters.

## D. 1 Not Insulated Steam Conduit

If there is no thermal insulation, such as in conduits for expansion the heat flow lost is [Positello, 1983]

$$
\begin{equation*}
\dot{Q}=\frac{\pi L(t i-t e)}{\frac{1}{K_{i} D_{i}}+\frac{1}{2 \lambda_{t}} \ln \left(\frac{D_{e t}}{D_{i}}\right)+\frac{1}{K_{e} D_{e}}} \tag{D.1}
\end{equation*}
$$

## D. 2 Insulated Steam Conduit

$$
\begin{equation*}
\dot{Q}=\frac{\pi L\left(T_{i}-T_{e}\right)}{\frac{1}{K_{i} D_{i}}+\frac{1}{2 \lambda_{t}} \ln \left(\frac{D_{e t}}{D_{i}}\right)+\frac{1}{2 \lambda_{C}} \ln \left(\frac{D_{e}}{D_{e t}}\right) \frac{1}{K_{e} D_{e}}} \tag{D.2}
\end{equation*}
$$

Where:
$\dot{Q}$ : heat loss $(W)$
$L$. Length of conduit, insulated or not (m)
$T_{i}$ : internal temperature conduit $\left({ }^{\circ} C\right)$
$T_{e}$ : external temperature ( ${ }^{\circ} \mathrm{C}$ )
$\pi: \mathrm{pi}=3.14159$
$K_{i}$ : heat transfer coefficient between internal fluid and wall of the conduit $\left(W /\left(m^{2}{ }^{\circ} C\right)\right)$
$D_{i}$ : interior diameter of tube ( $m$ )
$D_{e t}$ : external diameter of tube ( $m$ )
$\lambda_{t}$ : thermal conductivity of tube material $(W /(m . K))$
$K_{e}$ : heat transfer coefficient between external wall and the external fluid, here air $\left(W /\left(m^{2 \circ} C\right)\right)$
$\lambda_{C}$ : thermal conductivity of the insulating material (determined from curves) ( $W /(m \cdot K)$ )

## D. 3 Calculation of Heat Exchange Coefficients

## D.3.1 Superficial Heat Exchange Coefficient

The heat exchange coefficient $K_{i}$ depends on:

- Properties of the fluid (density, conductivity, specific heat, viscosity, and so on)
- Properties of wall surface (rugosity, possible pores, and so on)
$K_{i}$ is the superficial heat exchange coefficient between fluid and wall; it is the sum of exchange of convection $K_{i c}$ and exchange due to heat radiation $K_{i r}$ :

$$
\begin{equation*}
K_{i}=K_{i c}+K_{i r} \tag{D.3}
\end{equation*}
$$

In practice, $K_{i r}$ is often negligible.
For steam, $K_{i}$ is calculated by the formula of Mc Adams

$$
\begin{equation*}
K_{i}=0,024 \frac{\lambda_{f}}{D_{i}}\left(\frac{\rho V D_{i}}{\mu}\right)^{0,8} \tag{D.4}
\end{equation*}
$$

with:
$\lambda_{f}$ : Heat conduction coefficient of steam given by curves in function of temperature $(T)$ and pressure $(P)$. For example at $T=180^{\circ}$ and $P=$ 10bars, $\lambda_{f}=67.4 * 10^{-2} W /(\mathrm{mK})$.
$\rho$ : density of steam $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ given as function of temperature
$V$ : steam velocity ( $\mathrm{m} / \mathrm{s}$ )
$\mu$ : dynamic viscosity of fluid in Pa.s
$D_{i}$ : internal diameter ( $m$ ).

## D.3.2 External Heat Exchange Coefficient

$$
\begin{gather*}
K_{e}=K_{e c}+K_{e r}  \tag{D.5}\\
K_{e}=\frac{\lambda f}{D_{e}}\left(0,45+0,33\left(\frac{\rho v D_{e}}{\mu}\right)\right)^{0,56}+K_{e r} \tag{D.6}
\end{gather*}
$$

$$
\begin{equation*}
K_{e r}=R \frac{\left(T_{s e}^{4}-T_{e}^{4}\right)}{T_{s e}-T_{e}} \tag{D.7}
\end{equation*}
$$

Where:
$R$ : heat radiation coefficient of the sheet metal covering the insulation $\left(W /\left(m^{2} . K\right)\right.$. For a galvanized metal $R=1.59 * 10^{-8} W /\left(m^{2} . K\right)$.
$T_{e}$ : exterior or environment temperature,
$T_{s e}$ : temperature of exterior surface given approximately by:

$$
\begin{equation*}
T_{s e} \approx T_{e}+\frac{T_{i}-T_{e}}{10} \tag{D.8}
\end{equation*}
$$

For small diameters and calm air, one takes usually:

$$
\begin{equation*}
K_{e}=1,15\left(\frac{T_{s e}-T_{e}}{D_{e}}\right)^{\frac{1}{4}}+K_{e r} \tag{D.9}
\end{equation*}
$$

Remark 2 If the external temperature $T_{\text {se }}$ is known, we can calculate directly the losses by the relation:

$$
\begin{equation*}
\dot{Q}=K_{e} \pi L\left(T_{s e}-T_{e}\right) \tag{D.10}
\end{equation*}
$$

The coefficient $K_{e}$ is calculated from equation D.6. This formula allows to verify the value taken for the calculation of $K_{e}$.

## D. 4 References

[Positello, 1983] Positello, R., 1983, "La vapeur d'eau industrielle", Technique et documemntatiom, Lavoisier, Paris 1983.

## Appendix E

## Simulation Programs under Matlab-Simulink

## E. 1 Block Diagram for the Steam Accumulator



FIGURE E.1. Simulink block diagram of the steam Accumulator

## E. 2 Resolution of the equation of two phase mixture

\% calculation of pressure PGV (in bar) and quality of steam X (in \%) of $\%$ the mixture water-vapour. Given data are: specific volume \% Vmg (Vmg=Vgv/Mgv) in m3/kg and specific enthalpy hgv \% (hgv=Egv/Mgv) in $\mathrm{j} / \mathrm{kg}$ of the mixture function $y X P=X P g v(u)$
\% Inputs:
$\mathrm{hgv} 0=\mathrm{u}(2)^{*} 1 \mathrm{e}-3 ; \%$ enthalpy calculated by the model of the boiler in $\mathrm{j} / \mathrm{kg}$;
$\mathrm{VM}=0.175^{*} \mathrm{u}(1) ; \% 0.175 \mathrm{~m} 3$ is the geometric volume of the boiler
Pgvmem=u(3); \% pressure value calculated by the step before.
\%Thermodynamic functions
\% VMl, is the specific volume of liquid, VMv, specific volume of steam
$\% \mathrm{hl}$, enthalpy of liquid in $\mathrm{kJ} / \mathrm{kg}$ et hv , enthalpy of steam in $\mathrm{kJ} / \mathrm{kg}$
$\mathrm{VMl}=\left[\begin{array}{llll}0 & 0 & 0 & -3.59 \mathrm{e}-71.2456 \mathrm{e}-5 \\ 1.039 \mathrm{e}-3\end{array}\right] ;$
$\mathrm{VMv}=[-5.3 \mathrm{e}-52.07 \mathrm{e}-3$-3.20e-2 2.498e-1 -1.03 2.166];
$\mathrm{hl}=[0-2.43 \mathrm{e}-2$ 8.487e-1 -1.19e1 9.997e1 3.47e2];
$\mathrm{hv}=\left[\begin{array}{lllll}0 & 0 & 0 & -7.4 \mathrm{e}-1 & 1.721 \mathrm{e} 1 \\ 2.68 \mathrm{e} 3\end{array}\right]$;
$\%$ specific enthalpy of the mixture calculated by the model of the boiler hgv=[000000 hgv0];
\% specific volume of the the mixture ( $\mathrm{VMgv}=\mathrm{Vgv} / \mathrm{Mgv}$ )
VMgv=[ 0000000 VM$]$;
\% Resolution of the mixture equation
$\mathrm{V}=\operatorname{conv}((\mathrm{VMv}-\mathrm{VMl}),(\mathrm{hgv-hl}))-\operatorname{conv}((\mathrm{hv}-\mathrm{hl}),(\mathrm{VMgv-VMl}))$;
Pgv1=roots(V) \% pressure in the boiler in bars
\% Selection of positive and real roots
Pgvreel=zeros(size(Pgv1));
for $\mathrm{i}=1$ :length(Pgvreel);
if $\operatorname{imag}(\operatorname{Pgv} 1(\mathrm{i}))==0 \& 0<\operatorname{real}(\operatorname{Pgv} 1(\mathrm{i})) \operatorname{Pgvreel}(\mathrm{i})=\operatorname{Pgv1}(\mathrm{i})$
end; end
\%Selection of one roots
Pgv=0;
for $\mathrm{i}=1$ :length(Pgvreel);
if abs(Pgvreel(i)-Pgvmem)<abs(Pgv-Pgvmem) Pgv=Pgvreel(i);
end;end
$\mathrm{X}=((\mathrm{hgv}(6)-\mathrm{polyval}(\mathrm{hl}, \mathrm{Pg} v))) /(($ polyval(hv,Pgv)-polyval(hl,Pgv))$)$
yXP=[Pgv X 0]

## Appendix F

## Heat Radiation and Solar Energy

## F. 1 Units and Usual values of Power

A quantitative appreciation of usual power values is very useful to get a picture of our civilization. At the same time allows to estimate solar energy and also the so called glasshouse effect, both used in modern environmental discussions.

The unit of power is the Watt, as used universally for all powers and not only for electric power. This unit is combining with the prefixes milli, kilo, etc. to arrive at some usual powers.

1. A micro watt ( $\mu W$ or $10^{-6} W a t t$ ) is the consumption of an electronic wrist watch.
2. A milliwatt ( $m W$ or $10^{-3} W$ ) is the consumption of pilot lamps of electric appliances and computers.
3. A watt $(W)$ is the power of a flashlight (torch).
4. A kilowatt ( $k W$ or $10^{3} w a t t$ ) is the power of an electric heater.
5. A megawatt ( $M W$ or $10^{6}$ watt) is a Diesel locomotive.
6. A gigawatt ( $G W$ or $10^{9}$ watt) is the power of a nuclear electric power plant.
7. A terawatt ( $T W$ or $10^{12}$ watt) is the electric consumption of a small country.
8. A petawatt ( $P W$ or $10^{15}$ watt) has no direct signification, but $175 P W$ is the solar power that strikes the disc or projection of the earth.

Let us cite some intermediate values: the electric power installed in all power plants is about $360 T W$ for the USA, $70 T W$ for France and $8 T W$ for Switzerland. With locomotives, this $1 M W$ corresponds to relatively small ones, where one puts several in front of a train. Electric locomotives in Europe have more power, about $5 M W$, and the French high speed train ( $T G V$ ) still more, about $12 M W$.

## F. 2 Heat Radiation and Solar Energy

## F.2. 1 Solar Energy

The solar energy is almost $1.4 \mathrm{~kW} / \mathrm{m}$ which is the total solar power, divided by the interception by the earth, according to it's projected surface. The surface of the earth is:

$$
\begin{equation*}
S_{T}=\pi R_{T}^{2}=\pi * 6000 \mathrm{~m}^{2} \tag{F.1}
\end{equation*}
$$

This power comes to the surface at high altitude. A part is absorbed right away, so that approximately $1 \mathrm{~kW} / \mathrm{m}^{2}$ touches the earth at sea level on a bright day, much less if the sky is covered or foggy.

The efficiency of photocells is at best $20 \%$, one can draw about 200 W per $m^{2}$ on a clear day and if the cell is oriented towards the sun.

The entropy is calculated from the thermal energy, adding the factor $4 / 3$ (see equation F. 12 and F. 13 below). It is $38 T W / K$ for incoming radiation.

## F.2.2 Thermal Balance of the Earth

The thermal balance of the earth is shown on figure F.1. The same power as incoming is reradiated in the space in all directions, as shown. Most of this reflection takes place in the upper atmosphere, where the temperature is colder, about 250 K . Hence the entropy current leaving the earth is greater, about $950 T W / K . m^{2}$. Compared to this figure, the entropy generated, by humans or by decomposition in the nature, which is important, remains negligible.


FIGURE F.1. Thermal balance of the earth

## F.2.3 Thermal Radiation

After study of thermal radiation by conduction modelled by a simple $R$ Element and by convection, we introduce in this paragraph the thermal radiation. Let us remind that convection is expressed by a enthalpy balance and a coupled power bond.

In the transmission of heat by heat by radiation, the thermal transfer goes by electromagnetic vibrations which travel in a straight line without support of matter, contrary to conduction and convection. It consists of electromagnetic waves of a wave length of about 0.01 to $100 \mu \mathrm{~m}$, including the visible spectrum 0,38 to $0.76 \mu \mathrm{~m}$. Thermal radiation is important for solar energy and interesting because the equation of Carnot 3.1 must be modified. Thermal radiation requires both the thermal phenomena and the electric phenomena. They were studied at the end of the last century (1880-1902) by he great physicist Kirchhoff and Max Plank.

Let us consider a black cavity filled with electromagnetic waves and with a relative small hole for the admission and evacuation of radiation. The walls have a certain temperature and we want to find the energy and entropy in its function. Figure F. 2 shows this cavity and we must first distinguish between energy and energy divided by volume and also between entropy and entropy per volume.


FIGURE F.2. Radiation of a black body
We have therefore:

$$
\begin{align*}
& U=u V \\
& S=s V \tag{F.2}
\end{align*}
$$

The following derivation comes from [Pipard 1966] who has a second formulation in a foot note. Fuchs 1996, gives a longer derivation and speaks of the contents of the cavity as a photon gas which is the right way and it known since about the year 1900 .

It follows from the theory of electromagnetic radiation developed by Max Plank [Plank, 1923], that the waves send out by a black body exercise a
radiation pressure on the walls which is equal to one third of the energy density $u$.

$$
\begin{equation*}
P=\frac{u}{3} \tag{F.3}
\end{equation*}
$$

Let us take now the fundamental (or Gibbs) equation of a multiport $C$ in the usual form $d U$, but written with a parameter $t$, which could be the time

$$
\begin{equation*}
\frac{d U}{d t}=T \frac{d S}{d t}-P \frac{d V}{d t} \tag{F.4}
\end{equation*}
$$

The variables $U, S$, and $V$ are the total values, not the quantities per volume, which are designated by small letters. By inserting (F.2) into (F.4) one obtains:

$$
\begin{equation*}
V \frac{d u}{d t}+u \frac{d V}{d t}=T V \frac{d s}{d t}+T S \frac{d V}{d t}-P \frac{d V}{d t} \tag{F.5}
\end{equation*}
$$

from which one takes

$$
\begin{equation*}
\frac{d u}{d t}=T \frac{d s}{d t}+\left(T s-\frac{u}{3}+u\right) \frac{d V}{d t} \tag{F.6}
\end{equation*}
$$

Since the energy per volume must be independent of the volume, the expression inside the parenthesis must zero. This gives:

$$
\begin{align*}
& u=\frac{3}{4} T s \\
& s=\frac{4}{3} \frac{u}{T} \tag{F.7}
\end{align*}
$$

Considering the fundamental equation per volume:

$$
\begin{equation*}
d u=T d s=\frac{4}{3} \frac{u}{s} d s \tag{F.8}
\end{equation*}
$$

as a differential equation of $u$ as function of $s$, one has the solution

$$
\begin{equation*}
u=\text { cste.s } s^{\frac{4}{3}} \tag{F.9}
\end{equation*}
$$

which gives with equation F .8 the expression

$$
\begin{equation*}
s=a T^{3} \tag{F.10}
\end{equation*}
$$

where a is a constant not determined by our derivation. We use equation F. 8 still to determine the internal energy

$$
\begin{equation*}
u=\frac{3}{4} a T^{4} \tag{F.11}
\end{equation*}
$$

The equation F. 11 is the radiation law of Stephan Boltzmann of the total energy. In order to determine the distribution on the different frequencies there must take the formula of Planck which was at the same time the beginning of the quantum mechanics. The radiation that comes out by the small hole is simply volume density multiplied by the speed of light

$$
\begin{align*}
& \dot{S}=a c T^{3} \\
& \dot{Q}=\frac{3}{4} a c T^{4} \tag{F.12}
\end{align*}
$$

where $c$ is the speed of light. One deducts:

$$
\begin{align*}
& \dot{Q}=\frac{3}{4} T \dot{S} \\
& \dot{S}=\frac{4}{3} \frac{\dot{Q}}{T} \tag{F.13}
\end{align*}
$$

Equation F. 13 gives a relation between the power flux by radiation $\dot{Q}$ and the flux of entropy $\dot{S} .$. We can also say that the thermal radiation contains $133 \%$ of entropy compared to the $100 \%$ of entropy in thermal conduction.

## F.2. 4 Solar Collector

The figure F. 3 shows the principle of transmission and connection of solar energy

The corresponding word bond graph appears on figure F.3b. The cavity has a bond which collects entropy and heat which its purpose. However, it produces a reflection of energy according to its own temperature in all the hemisphere. Since the sun has a temperature $T_{1}$ if both temperatures are equal, $T_{1}=T_{2}$, the irradiation in the space is the same and there is nothing to collect. A collector must therefore have a lower temperature. The temperature of the sun is high, about $6000^{\circ} \mathrm{C}$, according to the method of Fuchs [Fuchs, 1996].


FIGURE F.3. Solar collector. a. Scheme, b. simplified word bond graph, c. pseudo bond graph model

The pseudo bond graph model is given by figure F.3c, where $R S, R_{\text {rad }}$ and $R_{C}$ denote respectively the thermal resistance of solar energy towards the earth, the resistance of reflection and the resistance of conduction. The multiport- $C$ associated gives the storing of energy by the collector.

## F.2.5 Glass House Effect and Thermal Death

On speaks much since the time of Clausius, about 1850, about the thermal death of the earth and cites the entropy as responsible. But this is independent of the notion of entropy and Carnot's equation 1.3. Simply, each process has some friction which produces heat (and entropy) and heats the world.

Therefore, according the analysis of thermal death, the earth will be submerged in an excess of heat and entropy. We see that this is not so, because all heat and entropy is sent to the space. Spaces has a temperature of only about $4 K$ and can absorb all heat all long as this is so. More
probably is the cold death of the earth by exhaustion of the sun, but this will not happen in the next two billions of years.

One speaks also much of the glass house effect, which means a heating of the planet because of to much $\mathrm{CO}_{2}$ in the atmosphere. This also is not worrying, because a slight increase of the temperature of the atmosphere say of $1^{\circ} \mathrm{C}$, would introduce a larger radiation of 2.8 PW into space by Planks radiation law or equation F.12.

What is more to worry, is a return to the ice age, of which the last one has terminated only 20 thousand years ago. By comparison, the dinosaurs have disappeared since about 63 million years.

## F. 3 Philosophical Questions

We are at the end of our book and would like to think about the method employed. It seems that we have returned to the method of energy used by Ernst Mach (1838-1916) and Walter Ostwald (1853-1932) mainly until the years 1895 and we use energy as fundamental quantity. Ostwald said himself, that the natural system of units in physics is length, time and energy, instead of length, time and mass used in the SI units. This seems right to us.

This energy method was later eclipsed by the atomic approach of Plank and Einstein. It started with the kinetic gas theory by Maxwell (1831 1879) and Boltzmann (1844-1906).

We have used the energy method and have introduced, just like in electricity, two factors in the flux of thermal energy: temperature and entropy. This a return to the ideas of Sadi Carnot (1796-1832). It is expressed by the equation of Carnot 1.3 which applies exactly to substances without movement. For matter in movement, it remains approximately true and is calculated in convection by the concept of enthalpy.

The entropy as fundamental thermal quantity is a partial return to the theory of "caloricum", fashionable around 1800. It is a substance without mass, like electric charge, which can be indefinitely generated by dissipation. This introduces the notion of time, because each dissipation is irreversible. On the contrary, the mechanics of mass points or celestial mechanics, and also the electrodynamics of Maxwell are reversible. So with the energy method, we do not have the philosophical difficulty to explain irreversibility by reversible process.

Atomic theory is universally accepted to day. The temperature is there explained as proportional to the square of the momentum (or speed) of each DOF (Degree Of Freedom). This formulation goes well with bond graphs with the notion of effort and flow and is one reason why we are so attracted by the bond graph method.

For conclusion on atomic and energetic approaches, we think that both
are valid methods of developing knowledge and one must use the one best suited for the problem at hand. Jean Thoma has discussed these questions with Gottlieb Falk (1923-1992). He wanted to unify both approaches in his mind, but deceased in 1992 before finishing his work.

## F. 4 References

[Fuchs, 1996] Fuchs, H. U., "The Dynamics of Heat", Springer Verlag, 1996.
[Pippard, 1966] Pippard, A., "Elements of Classical Thermodynamic", Cambridge University Press, 1966.
[Plank, 1923] Plank, M., "Theorie der Wärmestrahliung", Verlag Barth, Leipzig, 1923.

## 7

## Liste of Used Symbols

| Symbol | Designation | Unit |
| :---: | :---: | :---: |
| A | Area | $m^{2}$ |
| $A_{f}$ | Forward affinity | J/mole |
| $A_{r}$ | Reverse affinity | J/mole |
| $c_{p}$ | Specific heat per mass at constant pressure | $J /(\mathrm{K.kg})$ |
| $C_{p}$ | Global Specific heat at constant pressure | $J / K$ |
| $c_{v}$ | Specific heat per mass at constant volume | $j /(\mathrm{K} . \mathrm{kg})$ |
| $C_{v}$ | Global Specific heat at constant volume | $J / K$ |
| $D$ | Diameter | $m$ |
| $\dot{E}$ | Energy flow, power | W |
| $e$ | Thickness | $m$ |
| $H$ | Total enthalpy | $J$ |
| $\dot{H}$ | Enthalpy flow | W |
| $h$ | Specific enthalpy | $J / \mathrm{kg}$ |
| $i$ | Electric current | A |
| $K_{C}$ | Thermal conductance | $W / K$ |
| $l$ | Width | $m$ |
| $L$ | Length | $m$ |
| $\dot{m}$ | Mass flow | $\mathrm{kg} / \mathrm{s}$ |
| $m$ | Mass | kg |
| $n$ | Molar mass | - |
| $N$ | Level | $m$ |
| $P$ | Pressure | $P a$ |
| $P u$ | Power | W |
| $\dot{Q}$ | Thermal power by conduction | W |
| $q$ | Electric charge | Coulomb |
| $\dot{S}$ | Entropy flow | W |
| $S$ | Entropy | $J$ |
| $s$ | Entropy density | $J / \mathrm{kg}$ |


| Symbol | Designation | Unit |
| :--- | :--- | :--- |
| $T$ | Temperature | ${ }^{\circ} \mathrm{C}$ or K |
| $\dot{n}$ | Molar flow | $\mathrm{mole} / \mathrm{s}$ |
| $u$ | Electric voltage | V |
| $v$ | Volume per mass | $\mathrm{m}^{3} / \mathrm{kg}$ |
| $V$ | Volume flow | $\mathrm{m}^{3} / \mathrm{s}$ |
| $V$ | Volume | $\mathrm{m}^{3}$ |
| $X$ | Quality of steam | $[0-1]$ |
| $U$ | Internal energy | J |
| $\dot{U}$ | Internal energy flow | W |
| $x$ | Displacement | m |
| $\nu$ | Specific volume | $\mathrm{m}^{3} / \mathrm{kg}$ |
| $\mu$ | Chemical potential or tension | $\mathrm{J} / \mathrm{mole}$ |
| $\rho$ | Mass volumic (density) | $\mathrm{Kg} / \mathrm{m}^{3}$ |
| $\lambda$ | Thermal conductivity | $\mathrm{W} /(\mathrm{m} . \mathrm{K})$ |
| $\dot{\xi}$ | Speed of chemical reaction | $1 / \mathrm{s}$ |
| $\xi$ | Advancement of reaction | - |

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